Engineering Notebook

VOLUME 1

EE2203 Signals and System

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UNIT No 1

1 CONTINUOUS AND DISCRETE TIME SIGNALS

Q1. Show that Complex Exponential signal $x(t) = e^{jw0t}$ is periodic with period $\frac{2\pi}{w}$

Answer:

x(t) will be periodic if x(t+T)=x(t)

$$x(t+T) = x(t)$$
$$e^{jw0(t+T)} = e^{jw0t}$$
$$e^{jw0t}e^{jw0T} = e^{jw0t}$$
$$e^{jw0T} = 1$$
$$w0T = n(2\pi) - - - - - where - n = \text{int}\,eger$$
$$T = \frac{2\pi n}{w0}$$
$$Thus, the - fundamental - period = T = \frac{2\pi}{w0}$$

Q2. Examine whether the following signals are periodic or not? If periodic determine the fundamental period.

$1)\sin 12\pi t$

$2)\cos 2t + \sin\sqrt{3}t$

Answer:

$$x(t) = \sin 12\pi t$$
$$compare - with - \sin wt$$
$$w = 12\pi$$
$$or$$
$$T = \frac{2\pi}{w} = \frac{2\pi}{12\pi} = \frac{1}{6}$$

Since T is a ratio of two intergers, x(t) is periodic with fundamental period T= 1/6

$$\cos 2t + \sin\sqrt{3}t$$
$$x(t) = x1(t) = x2(t)$$
$$x1(t) = \cos 2t$$
$$x2(t) = \sin\sqrt{3}t$$
$$w1 = 2$$
$$2\pi f1 = 2$$
$$f1 = \frac{1}{\pi}$$
$$Therefore = T1 = \pi$$
$$x2(t) = \sin\sqrt{3}t$$
$$w2 = \sqrt{3}$$
$$2\pi f2 = \sqrt{3}$$
$$f2 = \frac{\sqrt{3}}{2\pi}$$
$$Therefore = T2 = \frac{2\pi}{\sqrt{3}}$$

Since T1/T2 is not a ratio of two integers, the given signal is non periodic

Q3. Examine whether the following signals are periodic or not? If periodic determine the fundamental period.

$1) e-|t|$

$2) \cos 4n$

Answer:

$1) e-|t|$

The plot of x(t) verses t does not repeat at all. So it is aperiodic.

$$2) \cos 4n$$
$$comparing-it-with-\cos 2\pi fn$$
$$2\pi f = 4$$
$$f = \frac{2}{\pi}$$

It is not rational number; x(n) is not periodic

Q4. Determine whether the following signals are energy signal or power signal? Calculate their energy or power.

$1) x(t) = u(t)$

Answer:

It is a non periodic signal extending from t=0 to t=∞ with amplitude remaining constant.

Hence it is a power signal with finite power and infinite energy

Then Normalized average power

$$P = \frac{Lim}{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$\frac{Lim}{T \to \infty} \frac{1}{2T} \int_{0}^{T} (1)^2 \, dt = \frac{1}{2}$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$\frac{Lim}{T \to \infty} \frac{1}{2T} \int_{0}^{T} (1)^2 \, dt = \infty$$

Here P=1/2 and E=∞ Hence it a power signal

Q5. Determine whether the following signals are energy signal or power signal? Calculate their energy or power.

$1) x(t) = e^{j[3t + (\frac{\pi}{2})]}$

Answer

The given signal is an infinite duration periodic signal which is a combination of sine and cosine signals. It can be a power signal

$$x(t) = e^{j[3t+(\frac{\pi}{2})]}$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} \left| e^{j[3t+(\frac{\pi}{2})]} \right|^2 dt$$

$$= \infty$$

$$P = \frac{Lim}{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left| e^{j[3t+(\frac{\pi}{2})]} \right|^2 dt$$

$$\frac{Lim}{T \to \infty} \frac{1}{2T} [2T] = 1$$

The average power of the signal is finite and the total energy of the signal is infinite

Therefore, the given signal is a power signal.

Q6. Determine the power and rms value of the following signal

Answer:

$$1)x(t) = 7\cos\left(20t + \frac{\pi}{2}\right)$$

Compare given equation with A cos(w0t+θ)

Then the power of the signal $P = \frac{7^2}{2} 24.5W$

$$rms - value = \sqrt{24.5}$$

$$2)x(t) = Ae^{j5t}$$

Given equation is equal to

X(t)=Acos5t+jAsin5t

Then the power of signal

$$\frac{A^2}{2} + \frac{A^2}{2} = A^2$$

$$rms - value - of - the - signal = \sqrt{A^2} = A$$

Q7. Determine which of the following signals are causal or non causal.

$$1)x(t) = e^{2t}u(t-1)$$

$$2)x(t) = 3\sin c2t$$

$$3)x(t) = 2u(-t)$$

Answer

$1) x(t) = e^{2t} u(t-1)$

The signal x(t) is causal because x(t)=0 for t<0

$2) x(t) = 3 \sin c 2t$

A sinc signal exists for t<0. So the given signal is non causal

$3) x(t) = 2u(-t)$

The given signal exists only for t<0. So the given signal is anti causal.

Q8. Determine whether the following signals are even or odd.

$1) x(t) = e^{-3t}$

$2) x(t) = u(t+2)$

Answer:

$1) x(t) = e^{-3t}$

$x(-t) = e^{3t}$

$-x(t) = -e^{-3t}$

Since x(-t) is not equal to x(t) and x(-t) is not equal to –x(t), the given signal is neither even signal nor odd signal

$$2) x(t) = u(t+2)$$
$$x(-t) = u(-t+2)$$
$$-x(t) = -u(t+2)$$
$$u(t+2) = \begin{cases} 1, \ for, t \geq -2 \\ 0, \ for, t < -2 \end{cases}$$
$$u(-t+2) = \begin{cases} 1, \ for, t \leq -2 \\ 0, \ for, t > 2 \end{cases}$$
$$-u(t+2) = \begin{cases} -1, \ for, t \geq -2 \\ 0, \ for, t < -2 \end{cases}$$

Since x(-t) is not equal to x(t) and x(-t) is not equal to –x(t), the given signal is neither even signal nor odd signal

Q9. Find the even and odd components of the following signals

Answer:

$$x(n) = \{5, 4, 3, 2, 1\}$$
$$n = 0, 1, 2, 3, 4$$
$$x(-n) = 1, 2, 3, 4, 5$$
$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$
$$= \frac{1}{2}[1, 2, 3, 4, 5 + 5, 4, 3, 2, 1]$$
$$= \{0.5, 1, 1.5, 2, 5, 2, 1.5, 1, 0.5\}$$
$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$
$$\frac{1}{2}[-1, -2, -3, -4, 0, 4, 3, 2, 1]$$
$$= \{-0.5, -1, -1.5, -2, 0, 2, 1.5, 1, 0.5\}$$

Q10. Define unit signum signal and sinc function.

Answer:

The unit signum function is define as

$$\operatorname{sgn}(t) = \begin{cases} 1, \; for, t \geq 0 \\ -1, \; for, t < 0 \end{cases}$$

It can be expressed in terms of unit step signal

Sgn(t)=-1+2u(t)

The sinc signal is define as

$$\sin c(t) = \frac{\sin t}{t}; \; for -\infty < t < \infty$$

The sinc function oscillates with period 2π and decays with increasing t. its value is zero at nπ, $n = \pm 1, \pm 2, \dots \dots \dots$ ...It is an even function of time.

References

[1] Alan V. Oppenheim, Alan S.Willsky, with S. Hamid 2nd Edition, 1996 Prentice Hall
[2] J. G. Proakis, D. G.Manolakis 3rd Edition, 1996 PHI

UNIT No 2

1 CONTINUOUS AND DISCRETE TIME SYSTEM

Q1.Determine whether the following equation are static or dynamic

$1) y(t) = x(t-3)$

$2) y(n) = x(n-2) + x(n)$

Answer

$1) y(t) = x(t-3)$

The output depends on past value of input. Therefore the system is dynamic.

$2) y(n) = x(n-2) + x(n)$

The system is described by difference equation. Therefore system is dynamic.

Q2. Check whether the following system are causal or not

$1) y(t) = x(2-t) + x(t-4)$

$2) y(n) = \sin[x(n)]$

Answer

$1) y(t) = x(2-t) + x(t-4)$

$for; t = -1; y(-1) = x(3) + x(-5)$

$for; t = 0; y(0) = x(2) + x(-4)$

$for; t = 1; y(1) = x(1) + x(-3)$

For some values of t, the output depends on the future input. Therefore the system is non-causal

Output depends on future values of input. Therefore the system is non-causal

Q3. Determine whether the following signals are linear or not.

$1) y(t) = e^{x(t)}$

Answer

$$
\begin{aligned}
&1) y(t) = e^{x(t)} \\
&y(t) = T[x(t)] = e^{x(t)} \\
&for-input-x1(t), y1(t) = e^{x1(t)} \\
&for-input-x2(t), y2(t) = e^{x2(t)} \\
&the-weighted-sum-of-output \\
&ay1(t) + by2(t) = ae^{x1(t)} + be^{x2(t)} \\
&the-output-due-to-weighted-sum-of-inputs \\
&y3(t) = T[ax1(t) + bx2(t)] = e^{[ax1(t)+bx2(t)]} \\
&y3(t) \neq ay1(t) + by2(t)
\end{aligned}
$$

The weighted sum of outputs is not equal to the output due to weighted sum of inputs. Superposition principle is not satisfied. So the system is non-linear.

Q4. Determine whether the following signals are linear or not.

1) $y(n) = x(n)\cos wn$

Answer

$$y(n) = x(n)\cos wn$$
$$y(n) = T[x(n)] = x(n)\cos wn$$
$$for - input - x1(n), y1(n) = x1(n)\cos wn$$
$$for - input - x2(n), y2(n) = x2(n)\cos wn$$
$$the - weighted - sum - of - output$$
$$ay1(n) + by2(n) = [ax1(n) + bx2(n)]\cos wn$$
$$the - output - due - to - weighted - sum - of - inputs$$
$$y3(n) = T[ax1(n) + bx2(n)] = [ax1(n) + bx2(n)]\cos wn$$
$$y3(n) = ay1(n) + by2(n)$$

The weighted sum of outputs is equal to the output due to weighted sum of inputs. Superposition principle is satisfied. So the system is linear.

Q5. Determine whether the following signals are time variant or invariant.

1) $y(t)=t^2x(t)$

Answer

1) $y(t)=t^2x(t)$

$$y(t)=T[x(t)]=t^2x(t)$$
$$output-due-to-input-delayed-by-T-\sec$$
$$y(t,T)=T[x(t-T)]=y(t)\Big|_{t=t-T}=(t)^2x(t-T)$$
$$output-delayed-by-T-\sec$$
$$y(t-T)=y(t)\Big|_{t=t-T}=(t-T)^2x(t-T)$$
$$y(t,T)\neq y(t-T)$$

Delayed output is not equal to the output due to delayed input. Therefore system is time-variant

Q6. Determine whether the following signals are time variant or invariant.

1) $y(n)=x(n)$

Answer

$$y(n)=T[x(n)]=x(n)$$
$$output-due-to-input-delayed-by-T-\sec$$
$$y(n,k)=T[x(n-k)]=y(n)\Big|_{x(n)=x(n-k)}=x(n-k)$$
$$output-delayed-by-T-\sec$$
$$y(n-k)=y(n)\Big|_{n=n-k}=x(n-k)$$
$$y(n,k)=y(n-k)$$

Delayed output is equal to the output due to delayed input. Therefore system is time-invariant

Q7. Determine whether the following system is static, linear, non-linear, causal non-causal
$y(n) = a^n u(n)$

Answer:

Static/Dynamic

The output at any instant depends only on the present values of input. Hence the system is static.

Linear/Non-Linear

$$for - x1(n)$$
$$y1(n) = a^n x1(n)$$
$$for - x2(n)$$
$$y2(n) = a^n x2(n)$$
$$weighted - sum - of - output$$
$$py1(n) + qy2(n) = pa^n x1(n) + qa^n x2(n) = a^n [px1(n) + qx2(n)]$$
$$output - due - to - weighted - sum - of - input$$
$$y3(n) = T[py1(n) + qy2(n)] = a^n [px1(n) + qx2(n)]$$
$$y3(n) = py1(n) + qy2(n)$$

Hence the system is linear.

Causal/non-Causal

The output depends only on present input it does not depends on future input.

Hence the system is causal.

Q8. For the system, $y(t) = x(t-5) - x(3-t)$ find linearity, time variant and causality of the system.

Answer

$y_1(t) = v(t-5) - v(3-t)$
$y_2(t) = k\,v(t-5) - k\,v(3-t) = ky_1(t)$
Let $x_1(t) = v(t)$, then $y_1(t) = v(t-5) - v(3-t)$
Let $x_2(t) = 2w(t)$, then $y_2(t) = w(t-5)-w(3-t)$
Let $x_3(t) = x(t) + w(t)$
Then, $y_3(t) = y_1(t) + y_2(t)$
Hence it is linear.
Again, $y_1(t) = v(t-5) - v(3-t)$
$\therefore y_2(t) = y_1(t-t_0)$
Hence, system is time-invariant
If $x(t)$ is bounded, then, $x(t-5)$ and $x(3-t)$ are also bounded, so stable system.
At $t=0$, $y(0) = x(-5)-x(3)$, therefore, the response at $t=0$ depends on the excitation at a later time $t=3$.
Therefore Non-Causal.

Q9 Determine whether the following digital system is BIBO stable or not?

$$y(n) = ax(n+1) + bx(n-1)$$

Answer

$$y(n) = ax(n+1) + bx(n-1)$$
$$if, x(n) = \delta(n)$$
$$Then, y(n) = h(n)$$
$$\text{Im}\, pulse - response$$
$$h(n) = a\delta(n+1) + b\delta(n-1)$$
$$when, n = 0, h(1) = a\delta(2) + b\delta(0) = b$$
$$when, n = 2, h(2) = a\delta(3) + b\delta(1) = 0$$
$$h(n) = \begin{cases} b, \ for, n = 1 \\ 0, otherwise \end{cases}$$
$$Therefore, \sum_{n=-\infty}^{\infty} |h(n)| = b$$
$$Necessary - and - sufficient - condition - for - stability$$
$$\sum_{n=-\infty}^{\infty} \begin{matrix} |h(n)| < \infty \\ so - the - system - is - BIBO - stable \end{matrix}$$

Q10 Explain Time Scaling operation.

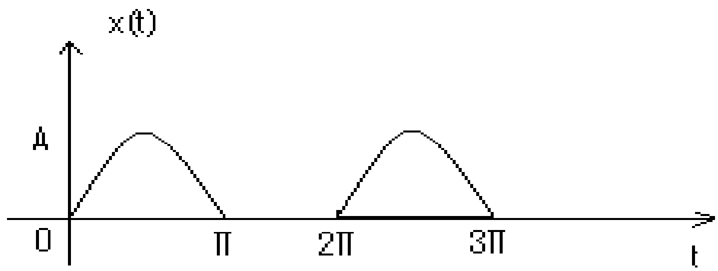
Answer

Time scaling may be time expansion or time compression. The time scaling of a signal x(t) can be accomplished by replacing t by at in it. Mathematically it can be expressed as y(t)=x(at). If a>1, it gives time compression by a factor a, if a<1, it gives expansion by a factor a because with that transformation a point at 'at' in signal x(t) becomes a point at 't' in y(t).

UNIT No 3

1 CONTINUOUS TIME FOURIER SERIES

Q1. Determine the trigonometric Fourier Series representation for half wave rectifier.



Answer:

$$x(t)=\begin{cases} A\sin wt = A\sin\frac{2\pi}{2\pi}t = A\sin t; 0\le t\le \pi \\ 0; \pi \le t \le 2\pi \end{cases}$$

$$fundamental-period-T=2\pi$$

$$Fundamental-frequency-w_0=\frac{2\pi}{T}=\frac{2\pi}{2\pi}=1$$

$$t_0=0, t_0+T=T=2\pi$$

$$a_0=\frac{1}{T}\int_0^T x(t)dt=\frac{1}{2\pi}\int_0^{2\pi}x(t)dt=\frac{1}{2\pi}\int_0^{\pi}A\sin tdt$$

$$=\frac{A}{2\pi}[-\cos t]_0^{\pi}=\frac{A}{2\pi}[-(\cos\pi-\cos 0)]_0^{\pi}$$

$$a_0=\frac{A}{\pi}$$

$$an=\frac{2}{T}\int_0^T x(t)\cos nw_0tdt=\frac{2}{2\pi}\int_0^{2\pi}x(t)\cos nw_0tdt$$

$$an=\frac{1}{\pi}\int_0^{\pi}A\sin t\cos ntdt=\frac{A}{\pi}\int_0^{\pi}\sin t\cos ntdt$$

$$an=-\frac{A}{\pi}[\frac{\cos(1+n)\pi-\cos 0}{1+n}+\frac{\cos(1-n)\pi-\cos 0}{1-n}]$$

$$Odd=an=0$$

$$Even=an=-\frac{2A}{\pi(n^2-1)}$$

$$bn=\frac{2}{T}\int_0^T x(t)\sin nw_0tdt=\frac{2}{2\pi}\int_0^{2\pi}x(t)\sin nw_0tdt$$

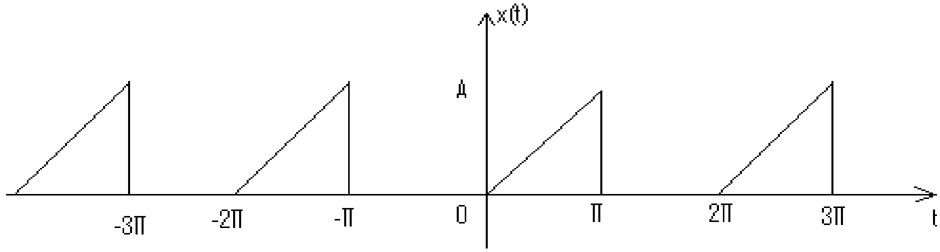
$$bn=\frac{1}{\pi}\int_0^{\pi}A\sin t\sin ntdt=\frac{A}{\pi}\int_0^{\pi}\sin t\sin ntdt$$

$$bn=\frac{A}{\pi}\int_0^{\pi}[\cos(n-1)t-\cos(n+1)t]dt$$

$$=\frac{A}{2\pi}\left[\frac{\sin(n-1)t}{n-1}-\frac{\sin(n+1)t}{n+1}\right]_0^{\pi}$$

$$bn=0$$

Q2 Determine the trigonometric Fourier Series representation for waveform shown below



Answer:

$$period = T = 2\pi$$

$$t_0 = 0, t_0 + T = 2\pi$$

$$w_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$$

$$x(t) = \begin{cases} (A/\pi)t;\ for 0 \le t \le \pi \\ 0; \pi \le t \le 2\pi \end{cases}$$

$$t_0 = 0, t_0 + T = T = 2\pi$$

$$a_0 = \frac{1}{T}\int_0^T x(t)dt = \frac{1}{2\pi}\int_0^{2\pi} x(t)dt = \frac{1}{2\pi}\int_0^{\pi} (A/\pi)tdt$$

$$= \frac{A}{4}$$

$$an = \frac{2}{T}\int_0^T x(t)\cos nw_0 tdt = \frac{2}{2\pi}\int_0^{\pi} \frac{A}{\pi} t\cos ntdt$$

$$= \frac{A}{\pi^2}\int_0^{\pi} t\cos ntdt$$

$$an = \frac{A}{\pi^2}\left[\left[\frac{t\sin nt}{n}\right]_0^{\pi} - \int_0^{\pi}\frac{\sin nt}{n}dt\right]$$

$$an = \frac{A}{\pi^2 n^2}\cos(n\pi - \cos 0)$$

$$Even = an = 0$$

$$Odd = an = -\frac{2A}{\pi^2 n^2}$$

$$bn = \frac{2}{T}\int_0^T x(t)\sin nw_0 tdt = \frac{1}{\pi}\int_0^{2\pi}\frac{A}{\pi} t\sin ntdt$$

$$bn = \frac{1}{\pi}\int_0^{\pi} A\sin t\sin ntdt = \frac{A}{\pi}\int_0^{\pi}\sin t\sin ntdt$$

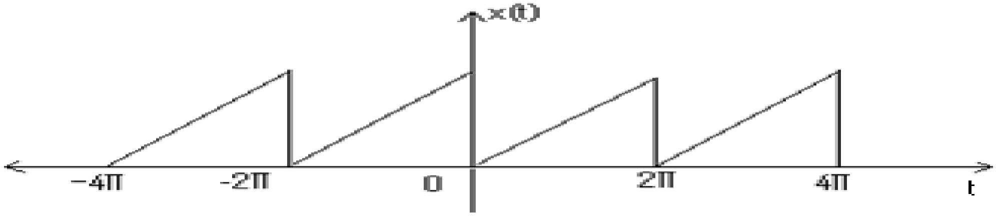
$$= \frac{A}{\pi^2}\left[-\pi\frac{\cos n\pi}{\pi} + \frac{\sin(n+1)t}{n^2}\right]_0^{\pi}$$

$$bn = A/n\pi\ for - odd - n$$

$$bn = -A/n\pi\ for - even - n$$

$$Therefore$$

Q3. Determine the trigonometric Fourier Series representation for waveform shown below



Answer:

$$x(t) = \frac{A}{2\pi} t, for\ 0 \leq t \leq 2\pi$$

t0=0

t0+T=T=2π

Fundamental frequency =$w0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$

Trigonometric series

$$x(t) = a0 + \sum_{n=1}^{\infty} an\ cos\ nw0t + bn \sin n\ w0t$$

$$x(t) = a0 + \sum_{n=1}^{\infty} an\ cos\ nt + bn \sin n\ t$$

$$a0 = \frac{1}{T} \int_0^T x(t) dt$$

$$a0 = \frac{1}{2\pi} \int_0^{2\pi} \frac{A}{2\pi} t dt$$

$$a0 = \frac{A}{2}$$

$$an = \frac{1}{T}\int_0^T x(t)\cos n\,wot\,dt$$

$$an = \frac{2}{2\pi}\int_0^{2\pi} (\frac{A}{2\pi}t)\cos n\,t\,dt$$

an=0

$$bn = \frac{1}{T}\int_0^T x(t)\sin n\,wot\,dt$$

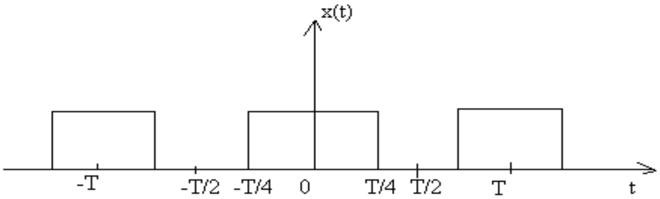
$$bn = \frac{2}{2\pi}\int_0^{2\pi} (\frac{A}{2\pi}t)\sin n\,t\,dt$$

$$bn = -\frac{A}{n\pi}$$

Trigonometric Fourier Series

$$x(t) = \frac{A}{2} + \sum_{n=1}^{\infty} -\frac{A}{n\pi}\sin n\,t$$

Q4. Determine the trigonometric Fourier Series representation for waveform shown below



Answer:

$$x(t) = A, for \ \frac{-T}{4} \leq t \leq \frac{T}{4}$$

$$= 0, for \ \frac{T}{4} \leq t \leq \frac{T}{2}$$

t0=-T/2

t0+T=T/2

Fundamental frequency =$w0 = \frac{2\pi}{T}$

Trigonometric series

$$x(t) = a0 + \sum_{n=1}^{\infty} an\ cos\ nw0t + bn \sin n\ w0t$$

$$x(t) = a0 + \sum_{n=1}^{\infty} an\ cos\ n\frac{2\pi}{T}t + bn \sin n\frac{2\pi}{T}\ t$$

$$a0 = \frac{1}{T}\int_0^T x(t)dt$$

$$a0 = \frac{1}{T}\int_0^{2\pi} Adt$$

$$a0 = \frac{A}{2}$$

$$an = \frac{1}{T}\int_0^T x(t)\cos n\, wot\; dt$$

$$an = \frac{2}{T}\int_0^{2\pi} (A)\cos n\; t\; dt$$

$$an = \frac{2A}{n\pi},\;\; for\; n = odd\; values$$

$$an = -\frac{2A}{n\pi},\;\; for\; n = even\; values$$

$$bn = \frac{1}{T}\int_0^T x(t)\sin n\, wot\; dt$$

$$bn = \frac{2}{T}\int_0^{2\pi} (A)\sin n\; t\; dt$$

$$bn = 0$$

Trigonometric Fourier Series

$$x(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \frac{2A}{n\pi} \sin\frac{n\pi}{2} cos\frac{n2\pi}{T} t$$

Q5. Find the complex exponential Fourier series representation of the following signals

1) $x(t) = 4cos2w0t$

Answer:

1) Given x(t)=4 cos2wot

$$x(t) = 4\left[\frac{e^{j2w0t} - e^{-j2w0t}}{2}\right]$$

$$x(t) = 2e^{j2w0t} + 2e^{-j2w0t}$$

$$Therefore\ above\ equation\ is\ the\ complex\ exponential\ Fourier\ series\ representation$$

Comparing this with complex exponential Fourier series

$$x(t) = \sum_{n=-\infty}^{\infty} Cne^{jnwot}$$

We get exponential Fourier Series coefficients

C2=2, C-2=2, Cn=0   for not equal to 0

Q6. Find the complex exponential Fourier series representation of the following signals

$$x(t) = 3sin4w0t$$

Answer:

1) Given x(t)=3 sin 4 w0 t

$$x(t) = 3\left[\frac{e^{j4w0t} - e^{-j4w0t}}{2j}\right]$$

$$x(t) = 3e^{j4w0t} + 3e^{-j4w0t}$$

$$Therefore\ above\ equation\ is\ the\ complex\ exponential\ Fourier\ series\ representation$$

Comparing this with complex exponential Fourier series

$$x(t) = \sum_{n=-\infty}^{\infty} Cne^{jnwot}$$

We get exponential Fourier Series coefficients

C4=3/2j, C-4=-3/2j , Cn=0   for not equal to 0

Q7. For the continuous time periodic signal x(t)=2+cos t +sin 4t, determine the fundamental frequency w0 and Fourier series coefficient Cn.

Answer:

$$x(t) = \sum_{n=-\infty}^{\infty} Cne^{jnwot}$$

Given x(t)=2+cos t +sin 4t

The time period of the signal cos 2t is :

T1=2π/2=π sec.

The time period of the signal sin 4t is :

T2=2π/4=π/2 sec.

T1/T2=2

The fundamental period of the signal x(t) is

T=T1=2T2=π sec

Fundamental frequency w0=2π/T = 2

x(t)=2+cos t +sin 4t

$$x(t) = 2 + \left[\frac{e^{j2t} + e^{-j2t}}{2}\right] + \left[\frac{e^{j4t} - e^{-j4t}}{2j}\right]$$

$= -\frac{1}{2j}e^{j4t} + \frac{1}{2}e^{-j2t} + 2 + \frac{1}{2}e^{j2t} + \frac{1}{2j}e^{j4t}2$

Exponential Fourier Coefficients C-2=-1/2j , C-1=1/2,

C0=2, C1= ½, C2= 1/2j

Q8. What are the Dirichlet's conditions? State them.

Answer:

The conditions under which the periodic signal can be represented by a Fourier series are known as Dirichlet,s conditions.

They are as follows

1) The function x(t) must be single valued function
2) The function x(t) has only a finite number of maxima and minima.
3) The function x(t) has a finite number of discontinuities.
4) The function x(t) is absolutely integrable over one period.

Q9. What do you mean by Gibbs Phenomenon?

Answer

Gibbs discovered that for a periodic signalwith discontinuities, if the signal is reconstructed by adding the Fourier series, overshoots appear around the edges. These overshoots decay outwards in a damped oscillatory manner away from the edges. This is called Gibbs Phenomenon.

Q10. Write short note on exponential Fourier series.

Answer:

The function x(t) is expressed as a weighted sum of the complex exponential functions. Although the trigonometric form and the cosine representation are the common forms of Fourier series, the complex exponential form is more general and usually more convenient and more compact. It finds extensive application in communication theory.

UNIT No 4

CONTINUOUS TIME FOURIER TRANSFORM

Q1. Determine the Fourier Transform of following signals

Answer:

$$x(t) = te^{-at}u(t)$$
$$F[e^{-at}u(t)] = \frac{1}{a + jw}$$
$$Signal, x(t) = e^{at}u(-t) - is - time - reversal - of [e^{-at}u(t)]$$
$$therefore,$$
$$F[e^{at}u(-t)] = \frac{1}{a - jw}$$
$$e^{at}u(-t) \leftrightarrow \frac{1}{a - jw}$$

Q2. Determine the Fourier Transform of following signals

Answer:

$$x(t) = e^{-2t}u(t-1)$$

$$F[e^{-2t}u(t)] = \frac{1}{2+jw}$$

$$by - time - shifting - property$$

$$F[e^{-2(t-1)}] = \frac{e^{-jw}}{2+jw}$$

$$e^{2}F[e^{-2t}u(t-1)] = \frac{e^{-jw}}{2+jw}$$

$$F[e^{-2t}u(t-1)] = \frac{e^{-(2+jw)}}{2+jw}$$

Q3. Determine inverse Fourier Transform of

$$X(jw) = \frac{jw}{(3+jw)^2}$$

Answer:

$$F[te^{-at}u(t)] = \frac{1}{(a+jw)^2}$$
$$F[te^{-3t}u(t)] = \frac{1}{(3+jw)^2}$$
$$let$$
$$te^{-3t}u(t) = x1(t)$$
$$then, \frac{1}{(3+jw)^2} = X1(jw)$$
$$we-know$$
$$F[\frac{d}{dt}x1(t)] = jwX1(jw)$$
$$F^{-1}[jwX1(jw)] = \frac{d}{dt}[te^{-3t}u(t)]$$

Q4. Determine inverse Fourier Transform of

$$X\left(jw\right) = e^{-|w|}$$

Answer:

$$F[e^{-|t|}] = \frac{2}{1+w^2}$$
$$use-dual-property$$
$$F[\frac{2}{1+t^2}] = 2\pi[e^{-|w|}]$$
$$F[e^{-|w|}] = \frac{1}{\pi(1+t^2)}$$

Q5. Determine Fourier transform of following non periodic Signal

Answer:

$$x(t) = \delta(t)$$

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$$

$$X(jw) = \int_{-\infty}^{\infty} \delta(t)e^{-jwt}dt$$

$$=1$$

$$F[\delta(t)] = 1$$

Q6. Determine Fourier transform of following non periodic

Signal

Answer:

$$x(t) = e^{-at}u(t)$$

$$X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt$$

$$X(jw) = \int_{-\infty}^{\infty} e^{-at} e^{-jwt} dt$$

$$X(jw) = \int_{-\infty}^{\infty} e^{-(a+jw)t} dt$$

$$X(jw) = \frac{1}{-(a+jw)} e^{-(a+jw)t} \Big|_0^{\infty}$$

$$X(jw) = \frac{1}{(a+jw)}$$

$$F[e^{-at}u(t)] = \frac{1}{(a+jw)}$$

Q7. Determine Fourier transform of following non periodic

Signal

$$x(t) = e^{-|t|}$$

Answer:

$$X(jw) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt$$

$$X(jw) = \int_{0}^{\infty} e^{-(1-jw)t}dt + \int_{0}^{\infty} e^{-(1+jw)t}dt$$

$$X(jw) = \frac{-1}{1-jw} e^{-(1-jw)t} \Big|_{0}^{\infty} + \frac{-1}{1+jw} e^{-(1+jw)t} \Big|_{0}^{\infty}$$

$$= \frac{1}{1+jw} + \frac{1}{1+jw}$$

$$= \frac{2}{1+w^2}$$

$$e^{-|t|} \leftrightarrow \frac{2}{1+w^2}$$

Q8. Determine frequency response of an LTI system described by the differential equation

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = 2x(t)$$

Answer:

Given

$$\frac{d^2y(t)}{dt^2}+5\frac{dy(t)}{dt}+6y(t)=2x(t)$$

$$Fourier-Transform-on-both-side$$

$$(jw)^2Y(jw)+5jwY(jw)+6Y(jw)=2X(jw)$$

$$Y(jw)[(jw)^2+5jw+6]=2X(jw)$$

$$The-\ frequency-response-is$$

$$H(jw)=\frac{Y(jw)}{X(jw)}=\frac{2}{(jw)^2+5jw+6}$$

Q9. Consider a causal LTI system with frequency response

$$H(jw)=\frac{1}{jw+2}$$

For a particular input x(t) this system is observed to produce the output

$$y(t)=e^{-2t}u(t)-e^{-3t}u(t)$$

Determine x(t).

Answer:

Given

$$H(jw) = \frac{1}{jw+2}$$

$$y(t) = e^{-2t}u(t) - e^{-3t}u(t)$$

$$apply - FT$$

$$Y(w) = \frac{1}{jw+2} - \frac{1}{jw+3}$$

$$= \frac{(jw+3)-(jw+2)}{(jw+2)(jw+3)}$$

$$= \frac{1}{(jw+2)(jw+3)}$$

$$we - know - that$$

$$Y(jw) = H(jw)X(jw)$$

$$X(jw) = \frac{Y(jw)}{H(jw)}$$

$$= \frac{\left(\frac{1}{(jw+2)(jw+3)}\right)}{\left(\frac{1}{jw+2}\right)}$$

$$= \left(\frac{1}{jw+3}\right)$$

$$therefore$$

$$x(t) = e^{-3t}u(t)$$

Q10. What is the condition for existence of Fourier Transform of a signal x(t).

Answer :

Fourier Transform of a signal x(t) exist if the signal x(t) is absolutely integrable. That is

$$\int_{-\infty}^{\infty} |x(t)| < \infty$$

UNIT No 5

CONTINUOUS TIME LAPLACE TRANSFORM

Q1. Prove that the signals $\begin{array}{l} 1)x(t) = e^{-at}u(t) \\ 2)x(t) = -e^{-at}u(-t) \end{array}$ have the same X(s) and differ only in ROC.

Answer:

$$x(t) = e^{-at}u(t)$$
$$L[x(t)] = X(s) = L[e^{-at}u(t)]$$

Given $$= \int_0^\infty e^{-at}u(t)e^{-st}dt = \int_0^\infty e^{-at}e^{-st}dt$$
$$= \int_0^\infty e^{-(s+a)t}dt = [\frac{e^{-(s+a)t}}{-(s+a)}]_{t=0}^{t=\infty}$$
$$= \frac{1}{s+a}$$

This integral converges if Re(s+a)>0, i.e. Re(s), $\sigma > -a, so - ROC; \sigma > -a$ *therefore*

$$L[e^{-at}u(t)] = \frac{1}{s+a}; ROC; \sigma > -a$$

$$x(t) = -e^{-at}u(-t)$$
$$u(-t) = \begin{cases} 1;\ for; t \le 0 \\ 0;\ for; t > 0 \end{cases}$$

$$L[x(t)] = X(s) = L[-e^{-at}u(-t)]$$
$$= \int_{-\infty}^{\infty} -e^{-at}u(-t)e^{-st}dt = \int_{-\infty}^{0} -e^{-at}1e^{-st}dt$$
$$= \int_{-\infty}^{0} -e^{-(s+a)t}dt = -[\frac{e^{(s+a)t}}{(s+a)}]_{t=0}^{t=\infty}$$
$$= \frac{1}{s+a}$$

This integral converges if Re(s+a) < 0, i.e. Re(s), $\sigma < -a, so - ROC; \sigma < -a$ *therefore*

$$L[-e^{-at}u(-t)] = \frac{1}{s+a}; ROC; \sigma < -a$$

Laplace transform of two signals are identical but their ROCs are different.

$x(t) = e^{-at}u(t)$ is causal and signal $x(t) = -e^{-at}u(-t)$ is a non causal signal.

Q2. How is Laplace transform useful in the analysis of LTD systems?

Answer.

The Laplace transform is a powerful mathematical technique used to convert the differential equation in time domain into algebraic equations in s- domain. For analysis, the system is expressed in terms of different equations. The differential equations are converted into algebraic equations using Laplace transform and the initial conditions are inserted. The result is converted into time domain by taking the inverse Laplace transform. It is a simple and systematic method which provides the complete solution in one stroke by taking into account the initial conditions in a natural way at the beginning of the process itself.

Q3. What is the unique characteristic of an exponential function?

Answer.

The unique characteristic of an exponential function is its integration as well as its differentiation results in the same original function.

Q4. What is region of convergence (ROC)?

Answer.

The set of point in the same s-plane for which the Laplace transform of x(t), i.e. the function X(s) converges is called the ROC (or the range of Re(s),i.e. the Laplace transform converges is known as the ROC).

Q5. Why there is one-to-one correspondence between only the one-sided Laplace transform and its inverse Laplace transform.

Answer.

In the one-sided Laplace transform, all the time function are assumed to be positive, but in the two-sided Laplace transform, the time function may be +v or –v. Hence, there is a one-to-one correspondence only between the one-sided Lapls

Q6. Find the Laplace transform of

$$f(t)=e^{3t}u(-t)+e^{t}u(t)$$

Answer

$$LT[e^{bt}u(-t)]=-\frac{1}{s-b}\,with-ROC-\mathrm{Re}(s)<b$$
$$LT[e^{at}u(t)]=-\frac{1}{s-a}\,with-ROC-\mathrm{Re}(s)>a$$
$$therefore$$
$$LT[e^{3t}u(-t)]=-\frac{1}{s-3}\,with-ROC-\mathrm{Re}(s)<3$$
$$LT[e^{t}u(-t)]=\frac{1}{s-1}\,with-ROC-\mathrm{Re}(s)>1$$
$$Combined-ROC$$
$$The-ROC;1<\mathrm{Re}(s)<3$$
$$F(s)=-\frac{1}{s-3}+\frac{1}{s-1}=\frac{-(s-1)+(s-3)}{(s-3)(s-1)}=\frac{-2}{(s-3)(s-1)}$$

Q7. If , $F(s)=\dfrac{(s+3)}{(s+1)(s+2)}$, find the inverse Laplace transform for -2<Re(s)<-1

Answer

Here, $F(s)=\dfrac{(s+3)}{(s+1)(s+2)}$ with ROC -2<Re(s)<-1

$$\frac{2}{s+1}+\frac{-1}{s+2}$$

The pole corresponding to s=-1 lies to the right of ROC and hence it contributes to negative time function.

On other hand, pole corresponding to s=-2lies to the left and hence it contributes to the positive time function

$$f(t) = -2e^{-t}u(-t) + [-1e^{-2t}u(t)] = -2e^{-t}u(-t) - e^{-2t}u(t)$$

Q8. If

$F(s) = \dfrac{(s+3)}{(s+1)(s+2)^2}$ find the inverse Laplace transform for -2<Re(s)<-1

Answer:

$$F(s) = \frac{(s+3)}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+1)^2}$$

$$A = 2; C = -1; B = -2$$

$$F(s) = \frac{2}{s+1} + \frac{-2}{s+2} + \frac{-1}{(s+2)^2}$$

$$F(s) = -2e^{-t}u(-t) - 2e^{-2t}u(t) - te^{-2t}u(t)$$

Q9. Determine Laplace transform of Impulse function and Ramp function

Answer:

The impulse function is obtained by differentiating unit step function u(t)

$$\delta(t)=\frac{du(t)}{dt}$$

$$LT[\delta(t)]=LT[\frac{du(t)}{dt}]=sF(s)-f(0)$$

$$initially-relaxed, f(0)=0$$

$$LT[\delta(t)]=sF(s)=sLT[u(t)]=s(\frac{1}{s})=1$$

The ramp function is given by

f(t)=t

if F(s) be the Laplace transform of f(t)

$$F(s)=LT[f(t)]=\int_0^\infty te^{-st}dt$$

$$F(s)=\frac{te^{-st}}{s}\Bigg|+\int_0^\infty te^{-st}dt=\frac{1}{s^2}$$

Q10. Use analysis equation and find Laplace transform of the signal

1) $x(t)=u(t-2)$

Answer: $L[u(t)]=\frac{1}{s}$

By using time shifting property ,

$$l[x(t-t0)] = e^{-st0}X(s)$$

$$L[u(t-2)] = \frac{e^{-2s}}{s}$$

$2)x(t) = t^2 e^{-2t} u(t\ \ )$

$$L[e^{-2t}u(t)] = \frac{1}{s+2}$$

using time differentiation in s domain property

$$L[(-t)^n x(t)] = \frac{d^n X(s)}{ds^n}$$

$For, n = 2$

$$L[t^2 x(t)] = \frac{d^2 X(s)}{ds^2}$$

$$L[t^2 e^{-2t} u(t)] = \frac{d^2}{ds^2}\left[\frac{1}{s+2}\right]$$

$$\frac{d}{ds}\left[\frac{-1}{(s+2)^2}\right]$$

$$\frac{2}{(s+2)^3}$$

UNIT No 6

Z TRANSFORM

Q1. Determine the Z transform and the ROC of the signal $x(n)=[3(2^n)-4(3^n)]u(n)$

Answer:

If the signals $\begin{aligned}&x1(n)=2^n u(n)\\&x2(n)=3^n u(n)\end{aligned}$

Then x(n) can be written as

$$\begin{aligned}
&x(n)=3x1(n)-4x2(n)\\
&ztransform-is\\
&X(Z)=3X1(z)-4X2(z)\\
&i.e.,---\alpha^n u(n)=\frac{1}{1-\alpha z^{-1}}---ROC:|z|>|\alpha|\\
&x1(n)=2^n u(n)\leftrightarrow X1(z)=\frac{1}{1-2z^{-1}};ROC:|z|>|2|\\
&x2(n)=3^n u(n)\leftrightarrow X1(z)=\frac{1}{1-3z^{-1}};ROC:|z|>|3|
\end{aligned}$$

the intersection of ROC of X1(z) and X2(z) is $|z|>|3|$

Thus overall Z transform is $X(z)=\frac{3}{1-2z^{-1}}-\frac{4}{1-3z^{-1}},ROC|Z|>3$

Q2. Determine Z transform of the following signals. Use Z transform properties

Answer:

$$x(n) = u(-n)$$

$$Ztransform - u(n) = \frac{z}{z-1} = \frac{1}{1-z^{-1}}; ROC; |z| < 1$$

$$time - reversal - property$$

$$Z[u(n-n)] = \left.\frac{z}{z-1}\right|_{z=(\frac{1}{z})} = \frac{\frac{1}{z}}{\frac{1}{z}-1} = \frac{1}{1-z} = -\frac{1}{z-1}; ROC; |z| < 1$$

Q3. Use time shifting property find Z transform of the following

Answer

$$x(n) = \alpha^{n-2}u(n-2)$$

$$z - transform - x(n) = \frac{z}{z-\alpha}; ROC; |z| > |\alpha|$$

$$time - shifting - property$$

$$Z\{x(n-m)\} = z^{-m}X(z)$$

$$therefore$$

$$Z[\alpha^{n-2}u(n-2)] = z^{-2}Z[\alpha^{n}u(n)] = z^{-2}\frac{z}{z-\alpha} = \frac{1}{z(z-\alpha)}; ROC; |z| > |\alpha|$$

Q4 Determine Z transform of the following signal using convolution property

Answer

Z{x1(n)*x2(n)}=X1(z)X2(z) which implies that

X1(n)*x2(n)=Z^{-1} [X1(z)X2(z)]

$$X1(z) = 1 + 2z^{-1} - z^{-2} + 3z^{-4}$$
$$X2(z) = 1 + 2z^{-1} - z^{-2}$$
$$X1(z)X2(z) = (1 + 2z^{-1} - z^{-2} + 3z^{-4})(1 + 2z^{-1} - z^{-2})$$
$$x(n) = \{1, 4, 2, -4, 4, 6, -3\}$$

Q5. Find the inverse of Z transform of the following

Answer:

$$X(z) = \frac{\frac{1}{4}z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{4}z^{-1})}, ROC; |z| > \frac{1}{2}$$

On multiplying numerator and denominator with z^{-2}

$$X(z) = \frac{\frac{1}{4}z}{(z - \frac{1}{2})(z - \frac{1}{4})}$$

Above equation can be expressed in partial fraction

$$\frac{X(z)}{z}=\frac{c1}{z-\frac{1}{2}}+\frac{c2}{z-\frac{1}{4}}$$

Where c1 and c2 can be evaluated by partial fraction expansion equation

C1=1

C2=-1

$$\frac{X(z)}{z}=\frac{1}{z-\frac{1}{2}}-\frac{1}{z-\frac{1}{4}}$$

$$X(z)=\frac{z}{z-\frac{1}{2}}-\frac{z}{z-\frac{1}{4}}---ROC:|z|>\frac{1}{2}$$

Here sequence is causal and

$$x(n)=\left(\frac{1}{2}\right)^n u(n)-\left(\frac{1}{4}\right)^n u(n)$$

Q6. Determine the inverse of Z transform of the following

Answer:

$$X(z)=\frac{8z-9}{(z^2-5z+6)},ROC;|z|>3$$

$$\frac{X(z)}{z}=\frac{8z-19}{z(z^2-5z+6)}$$

$$\frac{X(z)}{z} = \frac{c1}{z} + \frac{c2}{z-2} + \frac{c3}{z-3}$$
$$\frac{-19}{6z} + \frac{3}{2(z-2)} + \frac{5}{3(z-3)}$$
$$X(z) = \frac{-19}{6} + \frac{3z}{2(z-2)} + \frac{5z}{3(z-3)}$$

On taking inverse Z transform

$$x(n) = \frac{-19}{6}\partial(n) + \frac{3}{2}(2)^n u(n) + \frac{5}{3}(3)^n u(n)$$

Q7 Determine all possible signals x(n) associated with z transform

Answer:

$$X(z) = \frac{5z^{-1}}{(1-2z^{-1})(1-3z^{-1})}$$

$$X(z) = \frac{5z^{-1}}{(1-2z^{-1})(1-3z^{-1})}$$
$$X(z) = \frac{5}{(z-2)(z-3)}$$
$$\frac{X(z)}{z} = \frac{5}{(z-2)(z-3)} = \frac{A}{z-2} + \frac{B}{z-3}$$
$$A = -5$$
$$B = 5$$
$$X(z) = \frac{-5z}{z-2} + \frac{5z}{z-3}$$

When ROC is Z>3 then signal x(n) is causal and all the terms are causal terms

$$x(n) = -5(2)^n u(n) + 5(3)^n u(n)$$

When the ROC is Z<2 the signal x(n) is anticausal and all the terms are anticausal

$$x(n) = 5(2)^n u(-n-1) - 5(3)^n u(-n-1)$$

When ROC is 2<Z<3 then the signal is two sided

$$x(n) = -5(2)^n u(n) - 5(3)^n u(-n-1)$$

Q8. What are the properties of region of Convergence?

Answer:

1. The ROC is a concentric ring or circular disc in the z plane centered at the origin
2. The ROC cannot contain any pole
3. The ROC of an LTI stable system contain the unit circle
4. The ROC must be connected region.

Q9. what are the different methods of evaluating inverse z transform?

Answer:

1. Long division method
2. Partial fraction expansion method

3. Residue method
4. Convolution method

Q10. Define system function

Answer

Let x(n) and y(n) are the input and output sequence of an LTI system with impulse h(n) . Then the system function of the LTI system is defined as the ratio of Y(z) and X(Z). That is

$$H(z) = \frac{Y(z)}{X(z)}$$

Where Y(z) is the z transform of the output signal y(n) and X(z) is the z transform of the input signal x(n).

Q11. What is Z transform of 1. Impulse signal 2. Unit step signal

Answer

1. Impulse signal

   $x(n) = \partial(n)$

   $z - transform - is - X(z) = 1$

2. For a unit step signal

   $u(n) = 1\{ for -- n \geq 0;$

   $u(n) = 0\{ for -- n < 0;$

   And the z transform is $X(z) = \sum_{n=0}^{\infty} 1.z^{-n} = \frac{1}{1-z^{-1}}$

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