Engineering Notebook

VOLUME 1

EE2207 Network Analysis

CONTRIBUTOR

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BOOK TITLE

CONTENTS

UNIT No 1

UNIT-1: **Nodal Analysis of Electric Circuits**
Basics of electric circuits, circuit elements and their voltage – current relationship, classification of circuit elements, sources - their types and characteristics, concept of equivalent sources, source transformation and duality, concept of supernode and V – shift, nodal analysis of circuits containing resistors, inductors, capacitors, transformers, and both independent and dependent sources to determine current, voltage, power, and energy.

1 NODAL ANALYSIS

Q1. Find voltages V_1 and V_2 of the network shown below using Nodal Analysis.

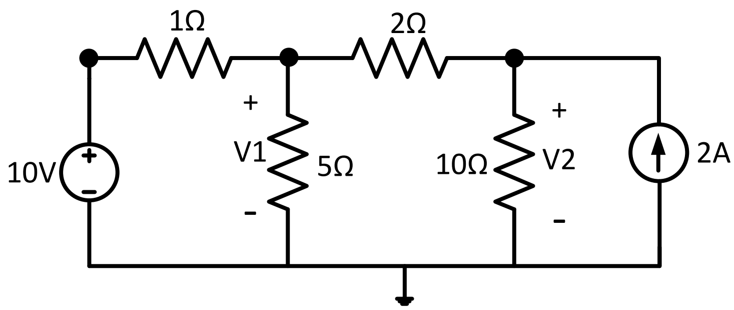

Figure: 1 Q1

Answer:

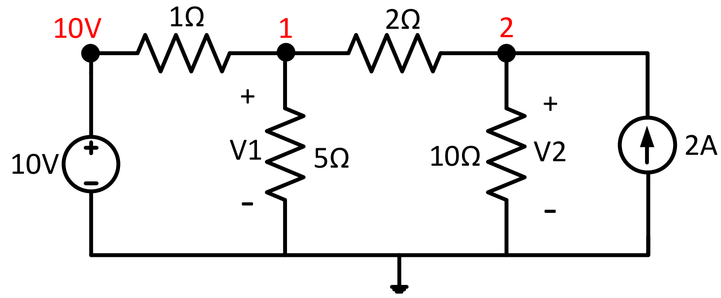

Figure: 2 Network with node numbers

There are 3 nodes in the above network say 1, 2, 3 but node 1 voltage is Known as =10V.
Again V1 and V2 is denoted in given network.
So KCL at node 1

$$\frac{V_1 - 10}{1} + \frac{V_1}{5} + \frac{V_1 - V_2}{2} = 0$$

$V_1 - 10 + 0.2\ V_1 + 0.5\ V_1 - 0.5\ V_2 = 0$

$1.7V_1 - 0.5V_2 = 10$ (1)

Then KCL at node 2

$$\frac{V_2}{10} + \frac{V_2 - V_1}{2} = 2$$

$0.1V_2 + 0.5V_2 - 0.5\ V_1 = 2$

$-0.5V_1 + 0.6V_2 = 2$ (2)

Equations (1) and (2) in matrix form

$$\begin{bmatrix} 1.7 & -0.5 \\ -0.5 & 0.6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$$

Solving above matrix

V_1= 9.0909V

V_2= 10.9090V

Q2.Determine the nodal voltages of the network shown below.

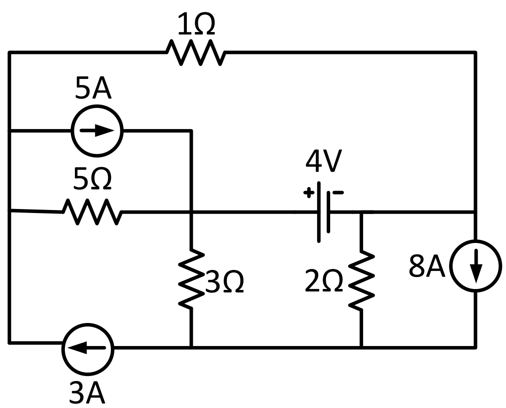

Figure: 3 Q2

Answer:

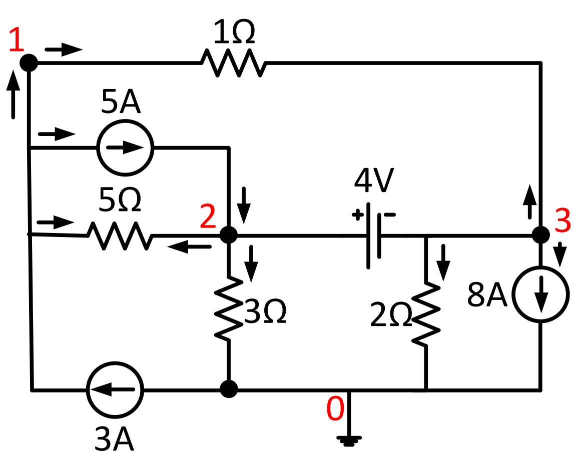

Figure: 4 Network with node numbers

There are 3 nodes in the given network 1, 2 and 3.

And nodal voltages V1, V2 and V3 respectively.

As an independent voltage source of 4V is connected between node 2 and 3 so they are forming a supernode.

KCL at node 1

$$\frac{V_1 - V_2}{5} + \frac{V_1 - V_3}{1} = -5 + 3$$

$0.2V_1 - 0.2V_2 + V_1 - V_3 = -2$

$1.2V_1 - 0.2V_2 - V_3 = -2$ (1)

KCL at supernode 2 and 3

$$\frac{V_2 - V_1}{5} + \frac{V_2}{3} + \frac{V_3}{2} + \frac{V_3 - V_1}{1} = 5 - 8$$

$0.2V_2 - 0.2V_1 + 0.33V_2 + 0.5V_3 + V_3 - V_1 = -3$

$-1.2V_1 + 0.53V_2 + 1.5V_3 = -3$ (2)

Equation from Supernode is

$V_2 - V_3 = 4$ (3)

Equations (1), (2) and (3) in matrix form

$$\begin{bmatrix} 1.2 & -0.2 & -1 \\ -1.2 & 0.53 & 1.5 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ 4 \end{bmatrix}$$

Solving above matrix

$V_1 = -8.61V$

$V_2 = -3.61V$

$V_3 = -7.61V$

Q3.Consider the circuit given below. Determine the current labelled $i1$ using nodal analysis.

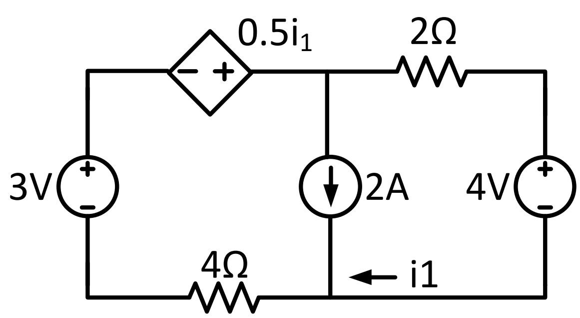

Figure: 5 Q3

Answer:

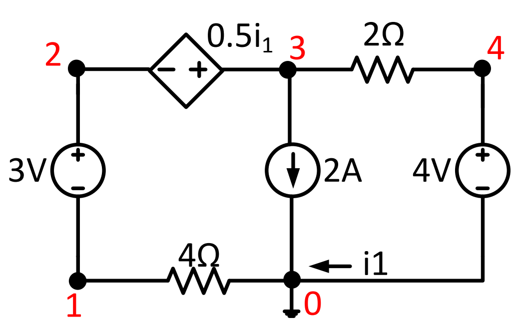

Figure: 6 Network with node numbers

There are total 5 nodes in the above network as 0,1,2,3 and 4.

V_0=0
And V_4=4V
As an independent voltage source of 3V is connected between node 1 and 2 & dependent voltage source of $0.5i_1$ is connected between node 2 and 3 so node 1 and 3 are forming a supernode.
KCL at supernode 1 and 3

$$\frac{V_1}{4} + \frac{V_3 - V_4}{2} = -2$$

$0.25V_1 + 0.5V_3 - 0.5V_4 = -2$
put $V_4 = 4V$
$0.25V_1 + 0.5V_3 - 2 = -2$
$0.25V_1 + 0.5V_3 = 0$ (1)
Equation between node 1 and 3
$V_3 - V_1 = 0.5i_1 + 3$
Where

$$i_1 = \frac{V_3 - V_4}{2} = \frac{V_3 - 4}{2}$$

Substituting in above equation

$$V_3 - V_1 = 0.5\frac{V_3 - 4}{2}$$

$V_3 - V_1 = 0.25V_3 - 1$
$V_3 - 0.25V_3 - V_1 = -1$
$0.75V_3 - V_1 = -1$
$-V_1 + 0.75V_3 = -1$ (2)
Equations (1) and (2) in matrix form

$$\begin{bmatrix} 0.25 & 0.5 \\ -1 & 0.75 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Solving this
V1=0.73V and V2= – 0.36V

Q4. Determine the nodal voltages of the network shown below using nodal analysis.

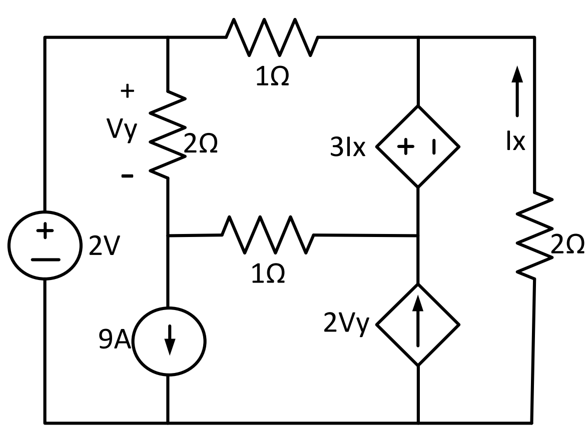

Figure: 7 Q4

Answer:

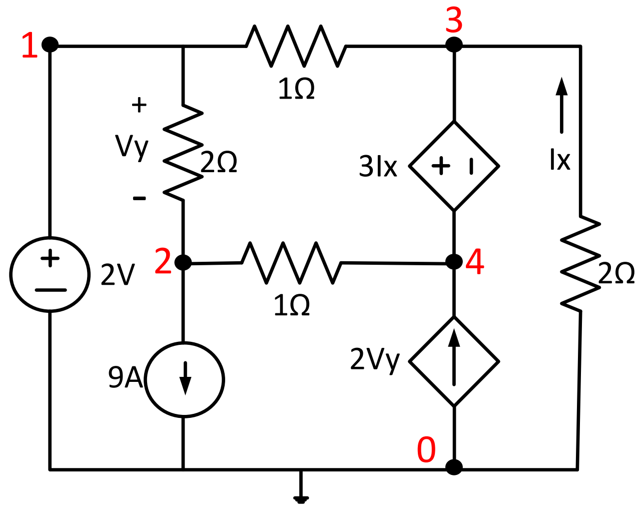

Figure: 8 Network with node numbers

Let v = 2v

KCL at node 1

$$\frac{v_1 - 2}{2} + \frac{v_1 - v_3}{1} = -9$$

$0.5v_1 - 1 + v_1 - v_3 = -9$

$1.5v_1 - v_3 = -8$-----------------------(1)

KCL at supernode 2 and 3

$$\frac{v_2 - 2}{1} + \frac{v_2}{2} + \frac{v_3 - v_1}{1} = 2v_y$$

Where $v_y = 2 - v_1$

$$\frac{v_2 - 2}{1} + \frac{v_2}{2} + \frac{v_3 - v_1}{1} = 2(2 - v_1)$$

$$\frac{v_2 - 2}{1} + \frac{v_2}{2} + \frac{v_3 - v_1}{1} = 4 - 2v_1$$

$v_2 - 2 + 0.5v_2 + v_3 - v_1 + 2v_1 = 4$

$v_1 + 1.5v_2 + v_3 = 6$-----------------------------(2)

supernode equation

$v_2 - v_3 = 3\ i_x$

where $i_x = -(v_2/2) = -0.5v2$

$v_2 - v_3 = 3-(v_2/2)$

$v_2 - v_3 = -1.5v_2$

$v_2 - v_3 + 1.5v_2 = 0$

$2.5v_2 - v_3 = 0$----------------------------(3)

Writing equation (1), (2) and (3) in matrix form

$$\begin{bmatrix} 1.5 & 0 & -1 \\ 1 & 1.5 & 1 \\ 0 & 2.5 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -8 \\ 6 \\ 0 \end{bmatrix}$$

Solving above matrix

V_1= -2V

V_2 = 2V

V_3 = 5V

Q5. Determine the nodal voltages of the network shown below using nodal analysis.

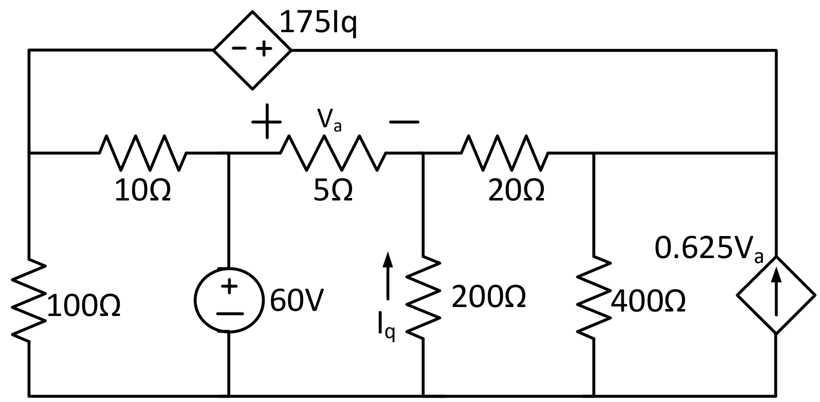

Figure: 9 Q5

Answer:

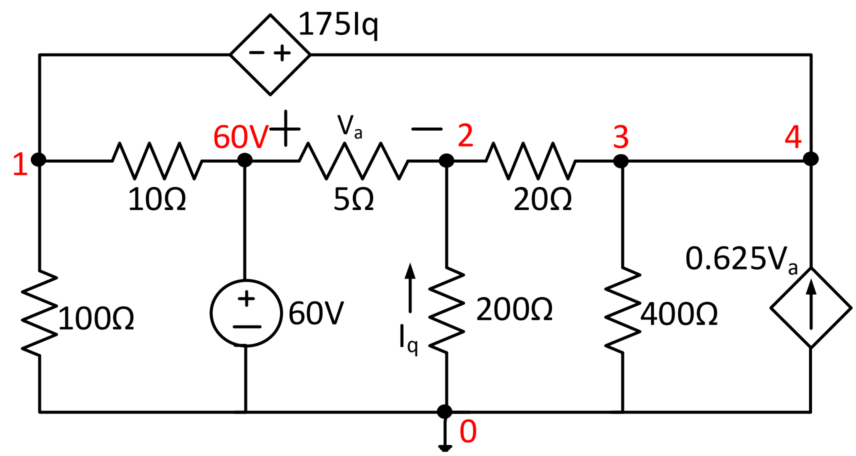

Figure: 10 Network with node numbers

KCL at supernode 1 and 3

$$\frac{v_1 - 60}{10} + \frac{v_1}{100} + \frac{v_3 - v_2}{20} + \frac{v_3}{400} = 0.625v_a$$

Where $v_a = v_2 - 60$

$$\frac{v_1 - 60}{10} + \frac{v_1}{100} + \frac{v_3 - v_2}{20} + \frac{v_3}{400} = 0.625(v_2 - 60)$$

$$\frac{v_1 - 60}{10} + \frac{v_1}{100} + \frac{v_3 - v_2}{20} + \frac{v_3}{400} = 0.625v_2 - 37.5$$

$0.1v_1 – 6 + 0.01v_1 + 0.05v_3 – 0.05v_2 + 0.0025v_3 – 0.625v_2 = –37.5$

$0.11v_1 – 0.675v_2 + 0.0525v_3 = – 31.5$------------------(1)

KCL at node 2

$$\frac{v_2 - 60}{5} + \frac{v_2 - v_3}{20} + \frac{v_2}{200} = 0$$

$0.2v_2 – 12 + 0.05v_2 – 0.05v_3 + 0.005v_2 = 0$

$0.255v_2 – 0.05v_3 = 12$-------------------------(2)

Equation from supernode

$v_3 – v_1 = 175\ I_q$

Where $I_q = – (v_2/200) = 175(– (v_2/200)) = –0.875v2$

$v_3 – v_1 = –0.875v_2$

$v_3 – v_1 + 0.875v_2 = 0$

$– v_1 + 0.875v_2 + v_3 = 0$-------------------------------(3)

Writing equation (1), (2) and (3) in matrix form

$$\begin{bmatrix} 0.11 & -0.675 & 0.0525 \\ 0 & .255 & -0.05 \\ -1 & .875 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -31.5 \\ 12 \\ 0 \end{bmatrix}$$

Solving above matrix

V_1 = -60.75V,

V_2 = 30V,

V_3 = -87V

Q6. Determine nodal voltages of the network shown below using nodal analysis.

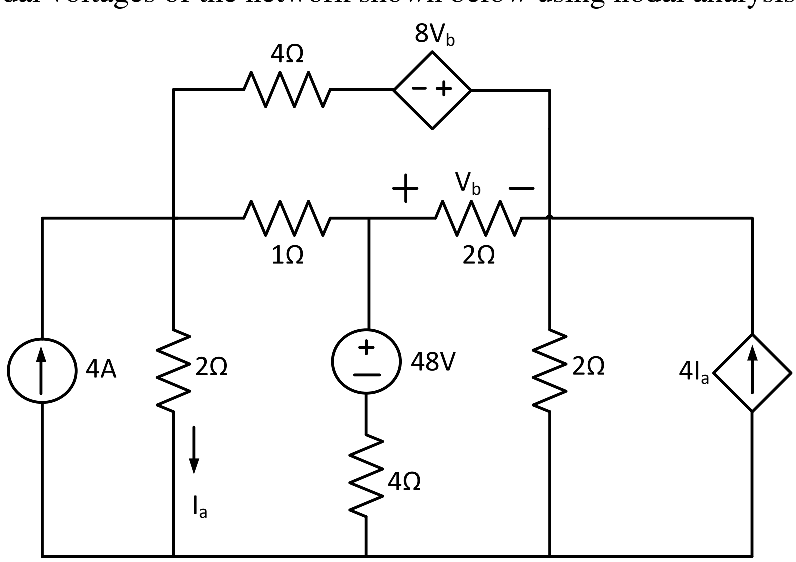

Figure: 11 Q6

Answer:

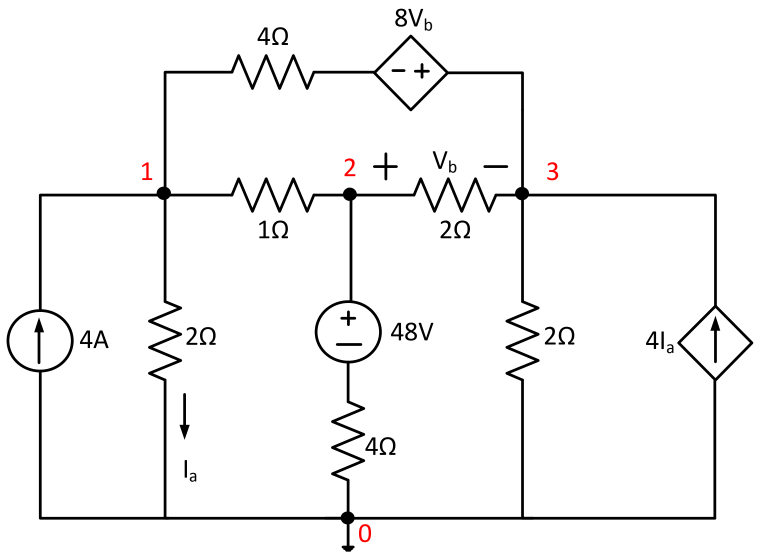

Figure: 12 Network with node numbers

From the network

$V_3 - V_1 = 8V_b$

Where $V_b = V_2 - V_3$

So

$V_3 - V_1 = 8(V_2 - V_3)$

$V_3 - V_1 = 8V_2 - 8\ V_3$

$V_3 - V_1 - 8V_2 + 8\ V_3 = 0$

$-V_1 - 8V_2 + 9V_3 = 0$

While writing for node 1

$V_1 + 8V_2 - 9V_3$

So KCL at node 1

$$\frac{V_1}{2} + \frac{V_1 - V_2}{1} + \frac{V_1 + 8V_2 - 9V_3}{4} = 4$$

$0.5V_1 + V_1 - V_2 + 0.25V_1 + 2V_2 - 2.25V_3 = 4$

$1.75V_1 + V_2 - 2.25V_3 = 4$---------------------------------------(1)

Again

$V_2 - 0 = 48$

$V_2 - 48 = 0$

KCL at node 2

$$\frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{2} + \frac{V_2 - 48}{4} = 0$$

$V_2 - V_1 + 0.5V_2 - 0.5V_3 + 0.25V_2 - 12 = 0$

$-V_1 + 1.75V_2 - 0.5V_3 = 12$-------------------------------------(2)

KCL at node 3

$$\frac{V_3 - V_2}{2} + \frac{V_3}{2} + \frac{9V_3 - V_1 - 8V_2}{4} = 4I_a$$

$0.5V_3 - 0.5V_2 + 0.5V_3 + 2.25V_3 - 0.25V_1 - 2V_2 = 4I_a$
$-0.25V_1 - 2.5V_2 + 3.25V_3 = 4I_a$
Where $I_a = \frac{V_1}{2} = 0.5V_1$
$-0.25V_1 - 2.5V_2 + 3.25V_3 = 4(0.5V_1)$
$-0.25V_1 - 2.5V_2 + 3.25V_3 = 2V_1$
$-0.25V_1 - 2.5V_2 + 3.25V_3 - 2V_1 = 0$
$-2.25V_1 - 2.5V_2 + 3.25V_3 = 0$---------------------------------(3)
Equations (1), (2) and (3) in matrix form

$$\begin{bmatrix} 1.75 & 1 & -2.25 \\ -1 & 1.75 & -.5 \\ -2.25 & -2.5 & 3.25 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 0 \end{bmatrix}$$

Solving above matrix
V1=-19.73V
V2=-10.67V
V3=-21.87V

Q7. Find the time-domain node voltages $v1(t)$ and $v2(t)$ in the circuit shown in Figure using nodal analysis.

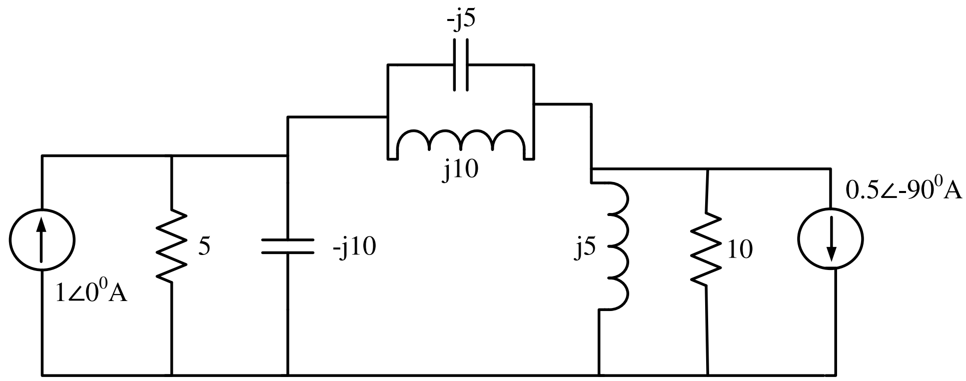

Figure: 13 Q7

Answer:

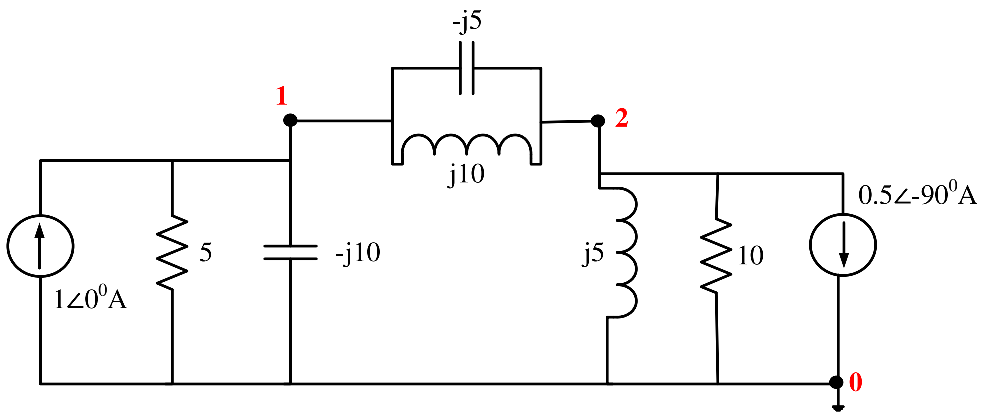

Figure: 14 Network with node numbers

KCL at node 1

$$\frac{V_1 - V_2}{-j5} + \frac{V_1 - V_2}{j10} + \frac{V_1}{5} + \frac{V_1}{-j10} = 1 \angle 0^0$$

$1 \angle 0^0 = 1$

$$V_1 \left[\frac{1}{-j5} + \frac{1}{j10} + \frac{1}{5} + \frac{1}{-j10}\right] + V_2 \left[-\frac{1}{-j5} - \frac{1}{j10}\right] = 1$$

$V_1\ (0.2 + 0.2j) + V_2(-0.1j) = 1$------------------------------------(1)

KCL at node 2

$$\frac{V_2 - V_1}{-j5} + \frac{V_2 - V_1}{j10} + \frac{V_2}{10} + \frac{V_2}{j5} = -(0.5 \angle -90^0)$$

$$V_1 \left[-\frac{1}{-j5} - \frac{1}{j10}\right] + V_2 \left[\frac{1}{-j5} + \frac{1}{j10} + \frac{1}{10} + \frac{1}{j5}\right] = 0.5j$$

$V_1(-0.1j) + V_2(0.1 - 0.1j) = 0.5j$----------------------------------(2)

Equations (1) and (2) in matrix form

$$\begin{bmatrix} 0.2 + 0.2j & -0.1j \\ -0.1j & 0.1 - 0.1j \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5j \end{bmatrix}$$

Solving above equations

$V_1 = 1 - j2$ V $= 2.24 \angle -63.4^0$ V

$V_2 = -2 + j4$ V $= 4.47 \angle 116.6^0$ V

$v_1(t) = 2.24 \cos(\omega t - 63.4^\circ)$ V

$v_2(t) = 4.47 \cos(\omega t + 116.6^\circ)$ V

Q8. Obtain V_0 of the network shown below using Nodal Analysis.

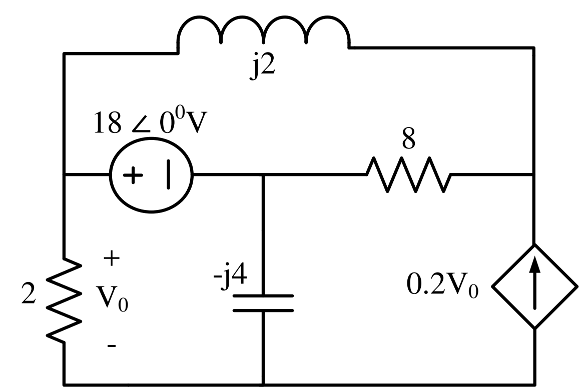

Figure: 15 Q8

Answer:

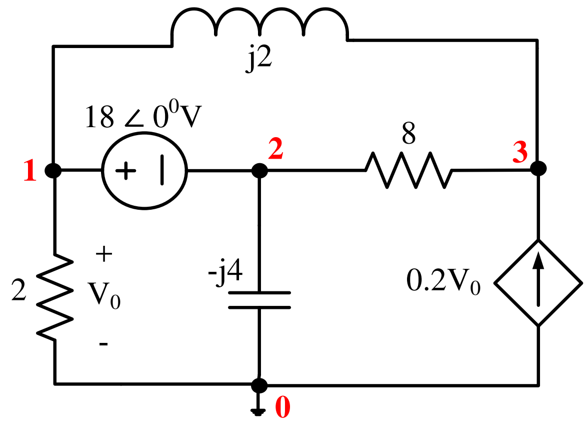

Figure: 16 Network with node numbers

There are 4 nodes in the given network as node 0, 1, 2 and 3.
And node 0 is a ground or reference or datum node.
As an independent voltage source is connected between node 1 and 2 so they are forming asupernode.
KCL at supernode 1 and 2

$$\frac{V_1 - V_3}{2j} + \frac{V_1}{2} + \frac{V_2}{-4j} + \frac{V_2 - V_3}{8} = 0$$

$$\frac{V_1}{2j} - \frac{V_3}{2j} + \frac{V_1}{2} + \frac{V_2}{-4j} + \frac{V_2}{8} - \frac{V_3}{8} = 0$$

$$V_1\left[\frac{1}{2j} + \frac{1}{2}\right] + V_2\left[\frac{1}{-4j} + \frac{1}{8}\right] + V_3\left[-\frac{1}{2j} - \frac{1}{8}\right] = 0$$

V_1 [0.5 – 05j] + V_2 [0.125 + 0.25j] + V_3 [–0.125 + 0.5j] = 0--------------------------------(1)

KCL at node 3

$$\frac{V_3 - V_2}{8} + \frac{V_3 - V_1}{2j} = 0.2V_0$$

Where $V_0 = V_1$

$$V_1[-\frac{1}{2j}] + V_2[-\frac{1}{8}] + V_3[\frac{1}{2j} + \frac{1}{8}] = 0.2V_1$$

V_1 [0.5j] + V_2 [–0.125] + V_3 [0.125 – 0.5j] = 0.2V1

V_1 [0.5j] – 0.2V_1 + V_2 [–0.125] + V_3 [0.125 – 0.5j] = 0

V_1 [0.5j – 0.2] + V_2 [–0.125] + V_3 [0.125 – 0.5j] = 0--------------------------(2)

Equation from supernode

$V_1 - V_2 = 18$ ---(3)

Equations (1), (2) and (3) in matrix form

$$\begin{bmatrix} 0.5 - 05j & 0.125 + 0.25j & -0.125 + 0.5j \\ 0.5j - 0.2 & -0.125 & 0.125 - 0.5j \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 18 \end{bmatrix}$$

$$\Delta=\begin{bmatrix} 0.5-05j & 0.125+0.25j & -0.125+0.5j \\ 0.5j-0.2 & -0.125 & 0.125-0.5j \\ 1 & -1 & 0 \end{bmatrix}=0.1625-0.1188i$$

$$\Delta_1=\begin{bmatrix} 0 & 0.125+0.25j & -0.125+0.5j \\ 0 & -0.125 & 0.125-0.5j \\ 18 & -1 & 0 \end{bmatrix}=2.2500+0.5625i$$

$$\Delta_2=\begin{bmatrix} 0.5-05j & 0 & -0.125+0.5j \\ 0.5j-0.2 & 0 & 0.125-0.5j \\ 1 & 18 & 0 \end{bmatrix}=-0.6750+2.7000i$$

$$\Delta_3=\begin{bmatrix} 0.5-05j & 0.125+0.25j & 0 \\ 0.5j-0.2 & -0.125 & 0 \\ 1 & -1 & 18 \end{bmatrix}=1.5750+0.9000i$$

$V_1 = \Delta_1 / \Delta = (7.3770 + 8.8525i)V = V_0$

$V_2 = \Delta_2 / \Delta = (-10.6230 + 8.8525i)V$

$V_3 = \Delta_3 / \Delta = (3.6798 + 8.2276i)V$

Q9. Use nodal analysis to determine the power supplied by the sources individually.

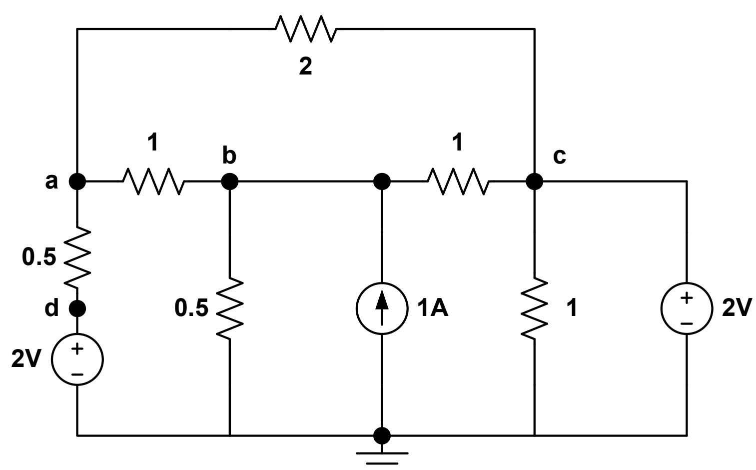

Figure: 17 Network with node numbers

Answer:

In the above network total 5 nodes are there. Out of that 1 is taken to be a reference node or ground node.

As the nodes are labeled in the given network a, b, c, d where $V_d = 2V$ and $V_c = 2V$ from given network

So we have write only 2 nodal equations at node a and b

KCL at node a

$$\frac{V_a-2}{0.5}+\frac{V_a-V_b}{1}+\frac{V_a-V_c}{2}=0$$

$$\frac{V_a}{0.5} - \frac{2}{0.5} + V_a - V_b + \frac{V_a}{2} - \frac{2}{2} = 0$$

$$\frac{V_a}{0.5} - \frac{2}{0.5} + V_a - V_b + \frac{V_a}{2} - 1 = 0$$

$$2V_a - 4 + V_a - V_b + 0.5V_a - 1 = 0$$

$$3.5V_a - V_b = 5 \qquad (1)$$

KCL at node b

$$\frac{V_b}{0.5} + \frac{V_b - V_a}{1} + \frac{V_b - V_c}{1} = 1A$$

$$\frac{V_b}{0.5} + V_b - V_a + V_b - 2 = 1$$

$$2V_b + V_b - V_a + V_b - 2 = 1$$

$$4V_b - V_a = 3$$

$$-V_a + 4V_b = 3 \qquad (2)$$

Writing equation (1) and (2) in matrix form as follows

$$\begin{bmatrix} 3.5 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$$

Δ=$\begin{bmatrix} 3.5 & -1 \\ -1 & 4 \end{bmatrix} = 14 - 1 = 13$

Δ_1=$\begin{bmatrix} 5 & -1 \\ 3 & 4 \end{bmatrix} = 20 + 3 = 23$

Δ_2=$\begin{bmatrix} 3.5 & 5 \\ -1 & 3 \end{bmatrix} = 10.5 + 5 = 15.5$

$V_a = \frac{\Delta_1}{\Delta} = \frac{23}{13}$ =1.7692V

$V_b = \frac{\Delta_2}{\Delta} = \frac{15.5}{13}$ =1.1923V

- Power supplied by 2V voltage source P = VI where I = V/R = V_a/0.5 = 1.7692/0.5 = 3.53
  - = 2*3.53
  - = 7.0768W

- Power supplied by 1A current source P = VI = V_b*I = 1.1923*1=1.1923W

- Power supplied by another 2V voltage source P = VI where I = V/R = V_c/1 = 2/1 = 2
- = 2*2 = 4W

Q10. Use nodal analysis to calculate the potentials V_A and V_B for the network shown in Fig.

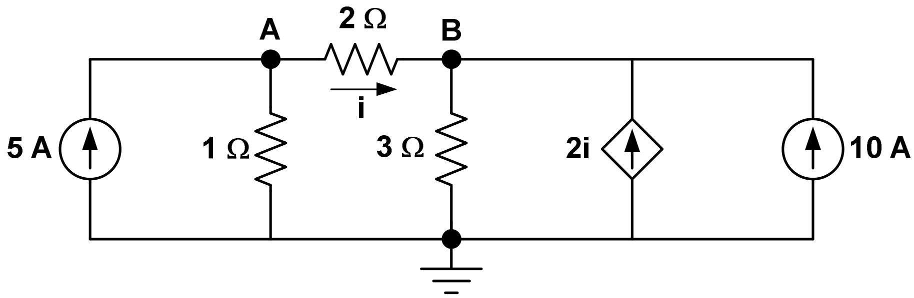

Figure: 18 Network with node numbers

Answer:

KCL at node A

$$\frac{V_A - V_B}{2} + \frac{V_A}{1} = 5A$$

$$0.5V_A - 0.5V_B + V_A = 5$$

$$1.5V_A - 0.5V_B = 5 \qquad (1)$$

KCL at node A

$$\frac{V_B - V_A}{2} + \frac{V_B}{3} = 10A + 2i$$

Where

$$i = \frac{V_A - V_B}{2}$$

Substituting in above equation

$$\frac{V_B - V_A}{2} + \frac{V_B}{3} = 10A + 2(\frac{V_A - V_B}{2})$$

$$0.5V_B - 0.5V_A + 0.33V_B = 10 + V_A - V_B$$

$$0.5V_B - 0.5V_A + 0.33V_B - V_A + V_B = 10$$

$$-1.5V_A + 1.83V_B = 10 \qquad (2)$$

Writing equation (1) and (2) in matrix form as follows

$$\begin{bmatrix} 1.5 & -0.5 \\ -1.5 & 1.83 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 1.5 & -0.5 \\ -1.5 & 1.83 \end{bmatrix} = 2.75 - 0.75 = 2$$

$$\Delta 1 = \begin{bmatrix} 5 & -0.5 \\ 10 & 1.83 \end{bmatrix} = 9.15 + 5 = 14.15$$

$$\Delta 2 = \begin{bmatrix} 1.5 & 5 \\ -1.5 & 10 \end{bmatrix} = 15 + 7.5 = 22.5$$

$$V_A = \frac{\Delta_1}{\Delta} = \frac{14.15}{2} = 7.075V$$

$$V_B = \frac{\Delta_2}{\Delta} = \frac{22.5}{2} = 11.25V$$

References

1. Network Analysis, 3rd Edition M. E. Van Valkenburg PHI Learning Private Limited.
2. Sudhakar, A., Shyammohan, S. P.; "Circuits and Network"; Tata McGraw-Hill New Delhi, 1994.
3. A William Hayt, "Engineering Circuit Analysis" 8th Edition, McGraw-Hill Education.
4. C. K. Alexander and M. N. O. Sadiku, "Fundamentals of Electric Circuits" 5th Edition, McGraw-Hill Education.

UNIT No 2

UNIT-2: **Mesh Analysis of Electric Circuits**
Concept of super mesh and I – shift, mutual inductance, coefficient of coupling, dot convention, dot marking in coupled coils, mesh analysis of circuits containing resistors, inductors, capacitors, transformers, and both independent and dependent sources to determine current, voltage, power, and energy.

1 MESH ANALYSIS

Q1. Find the mesh currents of the network shown below using mesh analysis.

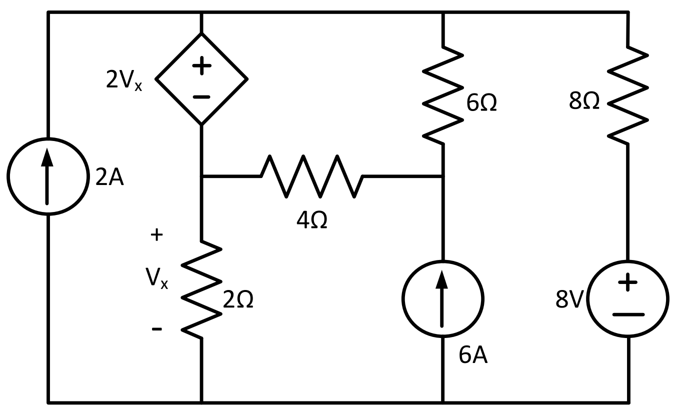

Figure: 19 Q1

Answer:

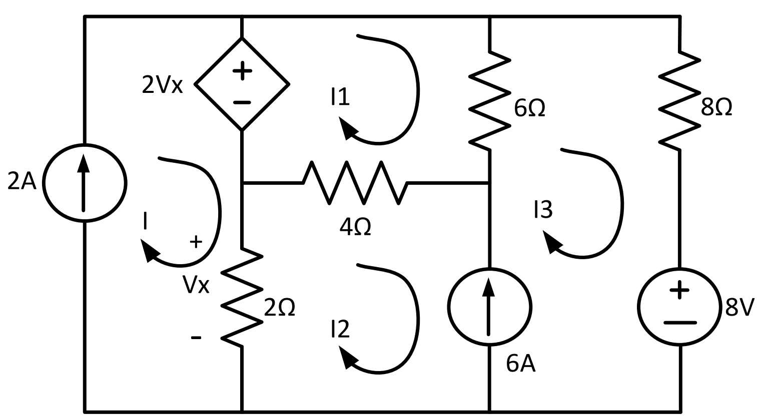

Figure: 20 Network with meshes

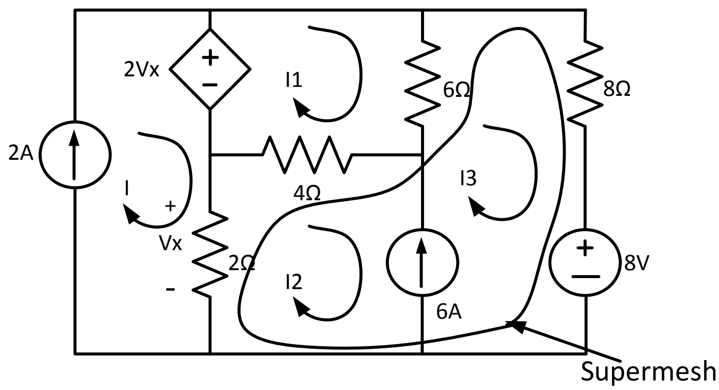

Figure: 21 Network with supermesh

Apparently there are 4 meshes in the above network as mesh 1, 2, 3 and 4.

But from the observation of the above network current of mesh 1 is 2A. i.e.I = 2A

KVL to mesh 1

$6(I_1 - I_3) + 4(I_1 - I_2) - 2V_X = 0$

where $V_X = 2\,(2 - I_2) = 4 - 2I_2$

$6(I_1 - I_3) + 4(I_1 - I_2) - 2(4 - 2I_2) = 0$

$6(I_1 - I_3) + 4(I_1 - I_2) - 8 + 4I_2 = 0$

$6I_1 - 6I_3 + 4I_1 - 4I_2 - 8 + 4I_2 = 0$

$10I_1 - 6I_3 = 8$---(1)

As an independent current source of 6A is connected between mesh 2 and 3 so these meshes are forming a supermesh.

KVL to supermesh 2 and 3

$2(I_2 - 2) + 4(I_2 - I_1) + 6(I_3 - I_1) + 8I_3 + 8 = 0$

$2I_2 - 4 + 4I_2 - 4I_1 + 6I_3 - 6I_1 + 8I_3 + 8 = 0$

$-10I_1 + 6I_2 + 14I_3 = -4$--(2)

Equation from supermesh

$I_3 - I_2 = 6$ --(3)

Equation (1), (2) and (3) in matrix form

$$\begin{bmatrix} 10 & 0 & -6 \\ -10 & 6 & 14 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -4 \\ 6 \end{bmatrix}$$

Solving above matrix

$I_1 = 2.857$A

$I_2 = 2.514$A

$I_3 = -3.143$A

Q2. Find the mesh currents of the network shown below using mesh analysis.

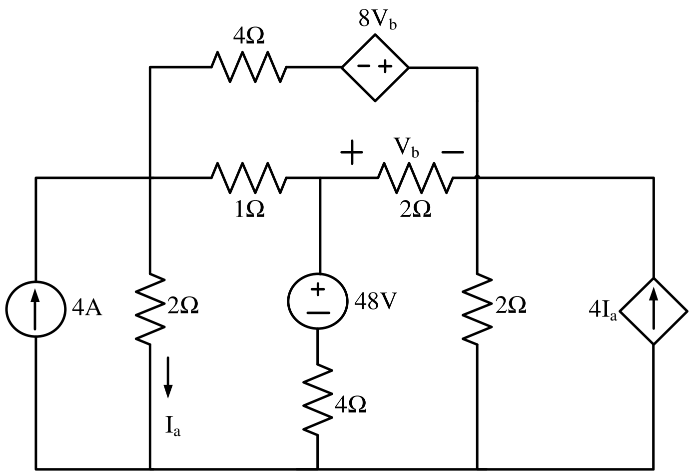

Figure: 22 Q2

Answer:

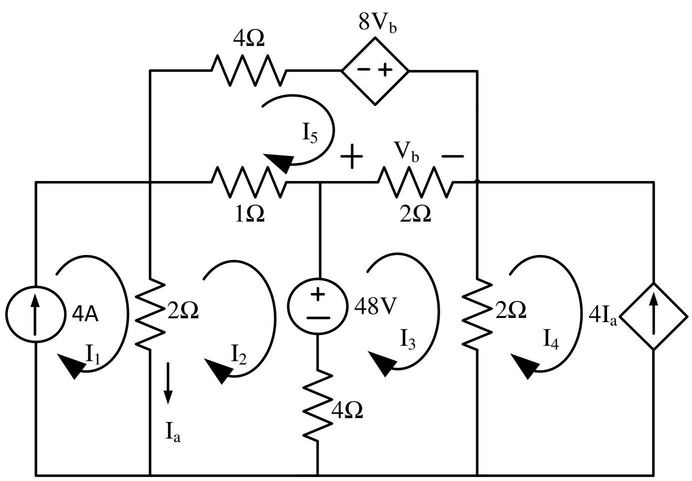

Figure: 23 Network with meshes

There are total 5 meshes in the given network.

Let the mesh current are i_1, i_2, i_3, i_4 and i_5

From the observation of network $i_1 = 4A$ and $i_4 = –4I_a$

KVL for mesh 2

$2(i_2 – 4) + 1(i_2 – i_5) + 48 + 4(i_2 – i_3) = 0$

$2i_2 – 8 + i_2 – i_5 + 48 + 4i_2 – 4i_3 = 0$

$7i_2 – 4i_3 – i_5 = – 40$----------------------------------(1)

KVL for mesh 2

$2(i_3 + 4i_a) + 2(i_3 – i_5) – 48 + 4(i_3 – i_2) = 0$

$2i_3 + 8i_a + 2i_3 – 2i_5 – 48 + 4i_3 – 4i_2 = 0$

Where $i_a = 4 – i_2$

$2i_3 + 8(4 – i_2) + 2i_3 – 2i_5 – 48 + 4i_3 – 4i_2 = 0$

$2i_3 + 32 – 8i_2 + 2i_3 – 2i_5 – 48 + 4i_3 – 4i_2 = 0$

$–12i_2 + 8i_3 – 2i_5 = 16$----------------------------(2)

KVL for mesh 5

$1(i_5 - i_2) + 2(i_5 - i_3) - 8V_b + 4i_5 = 0$

Where $V_b = 2(i_3 - i_5) = 2i_3 - 2i_5$

$i_5 - i_2 + 2i_5 - 2i_3 - 8(2i_3 - 2i_5) + 4i_5 = 0$

$i_5 - i_2 + 2i_5 - 2i_3 - 16i_3 + 16i_5 + 4i_5 = 0$

$- i_2 - 18i_3 + 23i_5 = 0$-----------------------------(3)

Equation (1), (2) and (3) in matrix form

$$\begin{bmatrix} 7 & -4 & -1 \\ -12 & 8 & -2 \\ -1 & -18 & 23 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -40 \\ 16 \\ 0 \end{bmatrix}$$

Solving above matrix

I_1=13.87A

I_2=28.53A

I_3=22.93A

Q3. Find the mesh currents of the network shown below using mesh analysis.

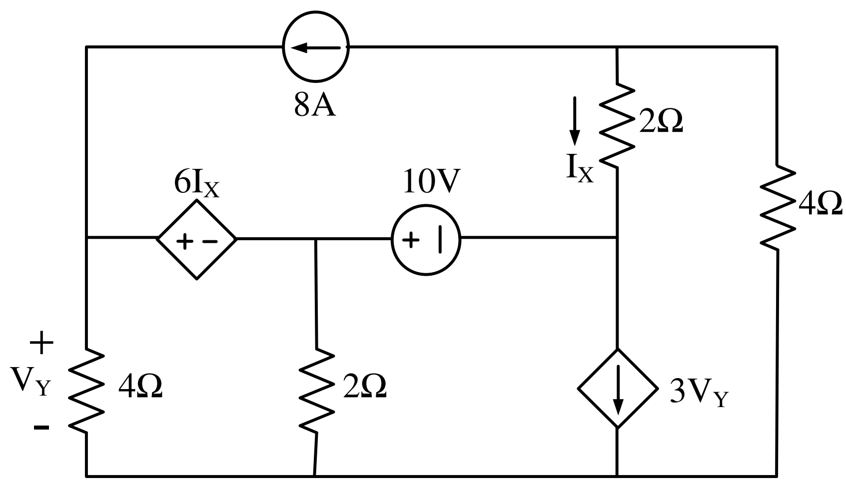

Figure: 24 Q3

Answer:

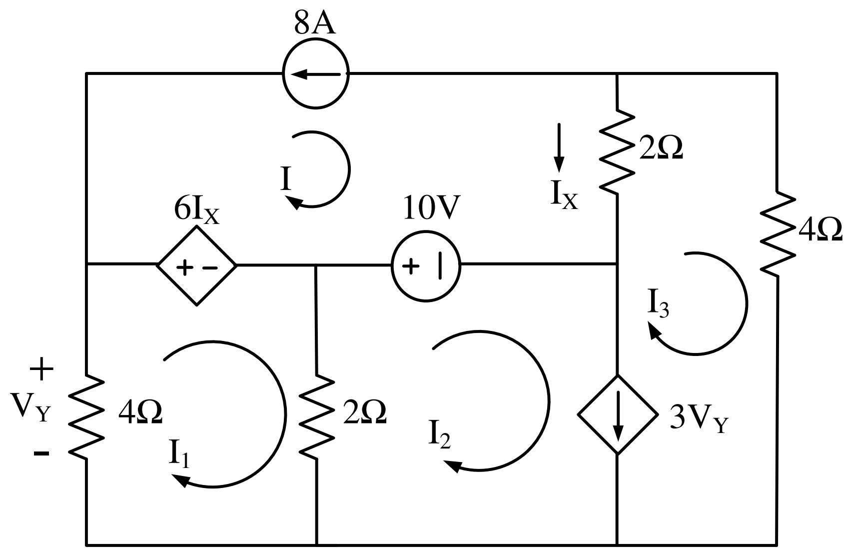

Figure: 23 Network with meshes

There are 4 meshes in the above network as mesh 1, 2, 3 and 4. But from the observation of given network one of the mesh current is know say current I = -8A as the direction of assumed mesh current i is exactly opposite to the independent current source of 8A connected in the same mesh.

So $I = -8$

KVL to mesh 1

$4I_1 + 2(I_1 - I_2) + 6I_X = 0$

Where $I_X = -8 - I_3$

$4I_1 + 2(I_1 - I_2) + 6(-8 - I_3) = 0$

$4I_1 + 2I_1 - 2I_2 - 48 - 6I_3 = 0$

$6I_1 - 2I_2 - 6I_3 = 48$------------------------------(1)

KVL to supermesh 2 and 3

$2(I_2 - I_1) + 10 + 2(I_3 + 8) + 4I_3 = 0$

$2I_2 - 2I_1 + 10 + 2I_3 + 16 + 4I_3 = 0$

$-2I_1 + 2I_2 + 6I_3 = -26$------------------------------(2)

Equation of supernode

$I_2 - I_3 = 3V_Y$

Where $V_Y = -4I_1$

$I_2 - I_3 = 3(-4I_1)$

$I_2 - I_3 = -12I_1$

$I_2 - I_3 + 12I_1 = 0$

$12I_1 + I_2 - I_3 = 0$ ------------------------------------(3)

Equation (1), (2) and (3) in matrix form

$$\begin{bmatrix} 6 & -2 & -6 \\ -2 & 2 & 6 \\ 12 & 1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 48 \\ -26 \\ 0 \end{bmatrix}$$

Solving above matrix

$I_1 = 5.5A$

$I_2 = -51.38A$

$I_3 = 14.62A$

Q4. Find the mesh currents of the network shown below using mesh analysis.

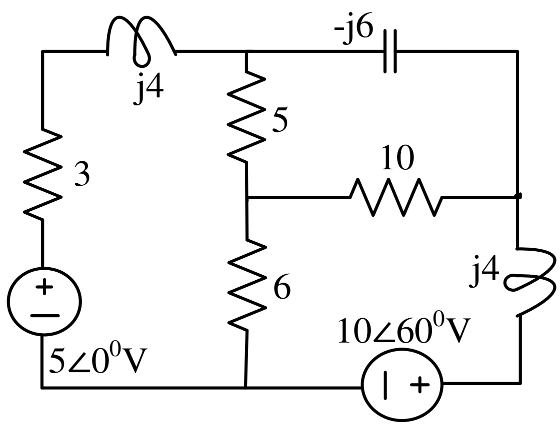

Figure: 24 Q4

Answer:

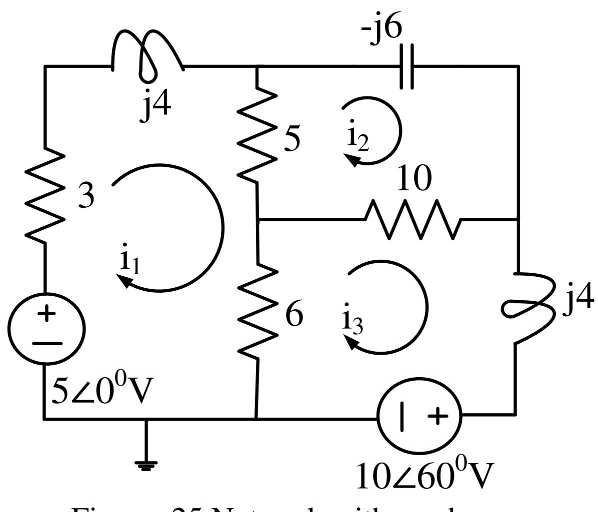

Figure: 25 Network with meshes

There are 3 meshes in the given network as meshs 1, 2 and 3.

KVL to mesh 1

$-5 \angle 0^0 + 3i_1 + j4\, i_1 + 5(i_1 - i_2) + 6(i_1 - i_3) = 0$

$-5\angle 0^0 + 3i_1 + j4\, i_1 + 5i_1 - 5i_2 + 6i_1 - 6i_3 = 0$
$(14 + j4)\, i_1 - 5i_2 - 6i_3 = 5$ ---------------------------------(1)

KVL to mesh 2
$5(i_2 - i_1) - j6i_2 + 10(i_2 - i_3) = 0$
$5i_2 - 5i_1 - j6i_2 + 10i_2 - 10i_3 = 0$
$-5i_1 + (15 - j6)i_2 - 10i_3 = 0$--------------------------------(2)

KVL to mesh 3
$10\angle 60^0 + 6(i_3 - i_1) + 10(i_3 - i_2) + j4i_3 = 0$
$6i_3 - 6i_1 + 10i_3 - 10i_2 + j4i_3 = -10\angle 60^0$
$-6i_1 - 10i_2 + (16 + j4)i_3 = -10\angle 60^0$ ---------------------(3)
Equation (1), (2) and (3) in matrix form

$$\begin{bmatrix} 14+j4 & -5 & -6 \\ -5 & 15-j6 & -10 \\ -6 & -10 & 16+j4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -(5+8.6602j) \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 14+j4 & -5 & -6 \\ -5 & 15-j6 & -10 \\ -6 & -10 & 16+j4 \end{bmatrix} = 900 + 268i$$

$$\Delta_1 = \begin{bmatrix} 5 & -5 & -6 \\ 0 & 15-j6 & -10 \\ -(5+8.6602j) & -10 & 16+j4 \end{bmatrix} = -191.77 - 1212.4i$$

$$\Delta_2 = \begin{bmatrix} 14+j4 & 5 & -6 \\ -5 & 0 & -10 \\ -6 & -(5+8.6602j) & 16+j4 \end{bmatrix} = 196.41 - 1572.2i$$

$$\Delta_3 = \begin{bmatrix} 14+j4 & -5 & 5 \\ -5 & 15-j6 & 0 \\ -6 & -10 & -(5+8.6602j) \end{bmatrix} = -552.84 - 1870.0i$$

Solving above matrix
$I_1 = \Delta_1/\Delta = (-0.5642 - 1.1791i)A = (1.3071\angle 115.57^0)A$
$I_2 = \Delta_2/\Delta = (-0.2774 - 1.6643i)A = (1.6872\angle -99.46^0)A$
$I_3 = \Delta_3/\Delta = (-1.1326 - 1.7405i)\,A = (2.0765\angle -123.05^0)A$

Q5. Obtain expressions for the time-domain currents i_1 and i_2 in the circuit given as Fig.

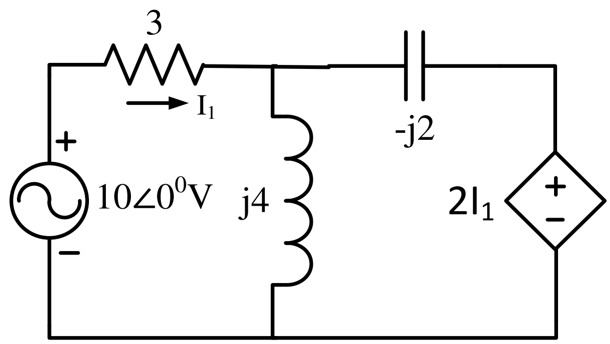

Figure: 26 Q5

Answer:

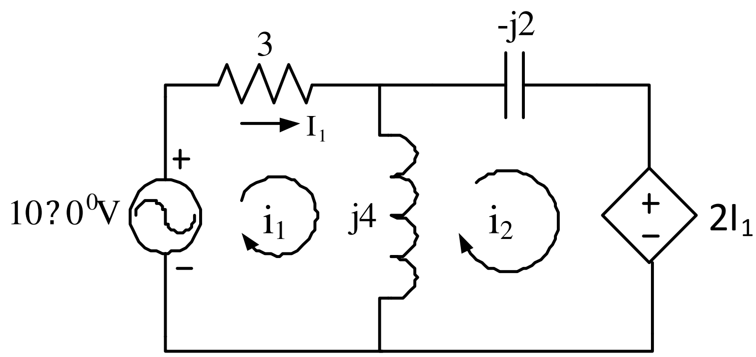

Figure: 27 Network with meshes

There are 2 meshes in the above network as 1 and 2. Therefore
KVL to mesh 1
$3I_1 + j4(I_1 - I_2) = 10\angle\ 0^0$
$(3 + j4)I_1 - j4I_2 = 10$----------------------(1)

KVL to mesh 2
$j4(I_2 - I_1) - j2I_2 + 2I_1 = 0$
$(2 - j4)I_1 + j2I_2 = 0$------------------------(2)

Equations (1) and (2) in matrix form
$$\begin{bmatrix} 3 + j4 & -4j \\ 2 - j4 & 2j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

Solving above equations
$I_1 = 1.0769 + 0.6153j$ A $= 1.24 \angle 29.74^0$ A
$I_2 = 1.5384 + 2.3076j$ A $= 2.77\angle\ 56.3^0$ A
$i_1(t) = 1.24 \cos(10^3t + 29.7^\circ)$ A
$i_2(t) = 2.77 \cos(10^3t + 56.3^\circ)$ A

Q6. Find mesh currents of the network shown below using mesh analysis.

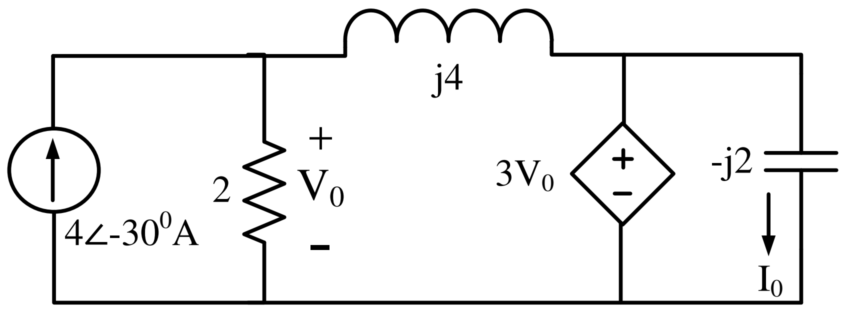

Figure: 28 Q6

Answer:

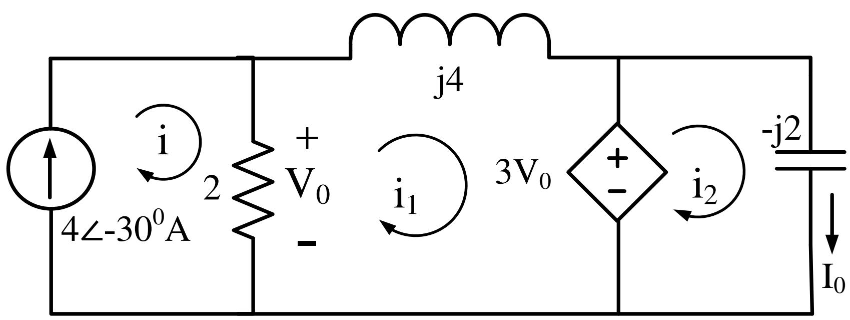

Figure: 29 Network with meshes

There are 3 meshes in the given network as 1, 2 and 3 and mesh currents are as i, i_2 and i_3 respectively. But from the observation of above network
$i = 4 \angle -30^0 = 3.4641 - 2j$

KVL to mesh 1
$2(i_1 - i) + j4i_1 + 3V_0 = 0$
$2(i_1 - (3.4641 - 2j)) + j4i_1 + 3V_0 = 0$
$2(i_1 - 3.4641 + 2j) + j4i_1 + 3V_0 = 0$
$2\,i_1 - 6.9282 + 4j + j4i_1 + 3V_0 = 0$
$(2 + j4)\,i_1 + 3V_0 = 6.9282 - 4j$
Where $V_0 = 2(i - i_1) = 2(3.4641 - 2j - i_1) = 6.9282 - 4j - 2i_1$
$(2 + j4)\,i_1 + 3(6.9282 - 4j - 2i_1) = 6.9282 - 4j$
$(2 + j4)\,i_1 + 20.7846 - 12j - 6i_1 = 6.9282 - 4j$
$(2 + j4)\,i_1 - 6i_1 = 6.9282 - 4j - 20.7846 + 12j$
$(-4 + 4j)\,i_1 = -13.8564 + 8j$
$i_1 = 2.7320 + 0.7320j$ A

Then $V_0 = 6.9282 - 4j - 2i_1 = 6.9282 - 4j - 2(2.7320 + 0.7320j)$
$V_0 = 6.9282 - 4j - 5.464 - 1.464j = 1.4642 - 5.464j$
KVL to mesh 2
$-3V_0 - 2ji_2 = 0$
$-3(1.4642 - 5.464j) - 2ji_2 = 0$
$-4.3926 + 16.392j - 2ji_2 = 0$
$-2ji_2 = 4.3926 - 16.392j$
$i_2 = 4.3923 - 16.3923j / -2j$
$i_2 = (8.1962 + 2.1962j)A$

Q7. Calculate $\mathbf{I}_1$ and $\mathbf{I}_2$ of the network shown below using mesh analysis.

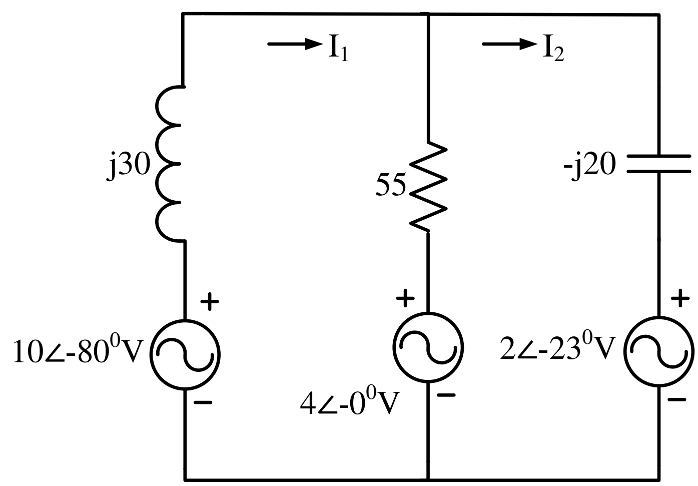

Figure: 30 Q7

Answer:

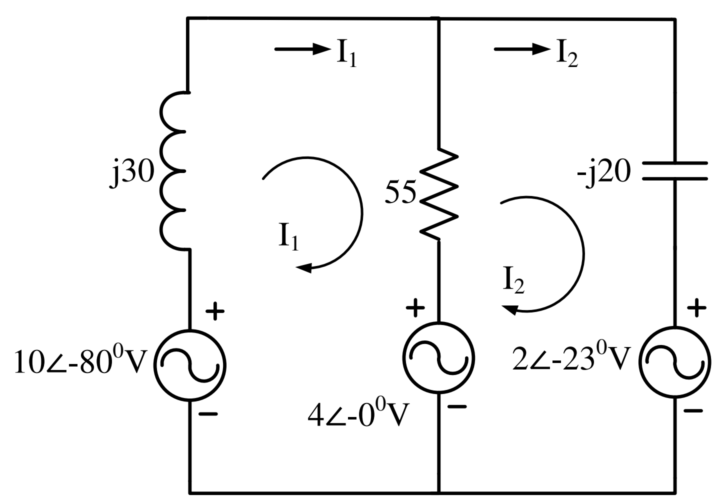

Figure: 31 Network with meshes

There are 2 meshes in the given network as mesh 1 and 2.
KVL to mesh 1
$-V_1 + 30j\, I_1 + 55(I_1 - I_2) + V_2 = 0$
Where given $V_1 = 10\angle{-80^0}$ V, $V_2 = 4\angle{-0^0}$ V
$-(10\angle{-80^0}) + 30j\, I_1 + 55(I_1 - I_2) + 4\angle{-0^0} = 0$
$-(1.7364 - 9.8480j) + 30j\, I_1 + 55(I_1 - I_2) + 4 = 0$
$30j\, I_1 + 55I_1 - 55\, I_2 = (1.7364 - 9.8480j) - 4$
$(30j + 55)I_1 - 55\, I_2 = -2.2636 - 9.848j$ ------------------------(1)
KVL to mesh 2
$-V_2 + 55(I_2 - I_1) - j20\, I_2 + V_3 = 0$
Where given $V_2 = 4\angle{-0^0}$ V are $V_3 = 2\angle{-23^0}$ V
$-(4\angle{-0^0}) + 55(I_2 - I_1) - j20\, I_2 + 2\angle{-23^0} = 0$
$-4 + 55\, I_2 - 55I_1 - j20\, I_2 + 1.8410 - 0.7814i = 0$
$-55I_1 + (55 - j20)\, I_2 = 4 - 1.8410 + 0.7814i$
$-55I_1 + (55 - j20)\, I_2 = 2.159 + 0.7814i$ ------------------------(2)
Equations (1) and (2) in matrix form

$$\begin{bmatrix} 30j + 55 & -55 \\ -55 & 55 - j20 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -2.2636 - 9.848j \\ 2.159 + 0.7814j \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 30j + 55 & -55 \\ -55 & 55 - j20 \end{bmatrix} = 600 + 550i$$

$$\Delta_1 = \begin{bmatrix} -2.2636 - 9.848j & -55 \\ 2.159 + 0.7814j & 55 - j20 \end{bmatrix} = -202.71 - 453.39i$$

$$\Delta_2 = \begin{bmatrix} 30j + 55 & -2.2636 - 9.848j \\ -55 & 2.159 + 0.7814j \end{bmatrix} = -29.195 - 433.89i$$

Solving above
$I_1 = -0.5600 - j0.2423 = 0.6092 \angle{-156.80^0}$ A
$I_2 = -0.3867 - 0.3687j = 0.5375 \angle{-136.51^0}$ A

Q8. Write mesh equilibrium equations for the circuit shown in Fig. and find the voltage V_X.

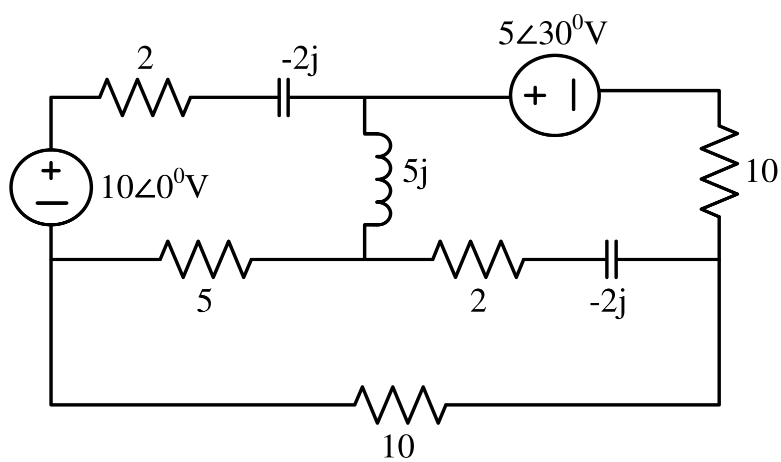

Figure: 32 Q8

Answer:

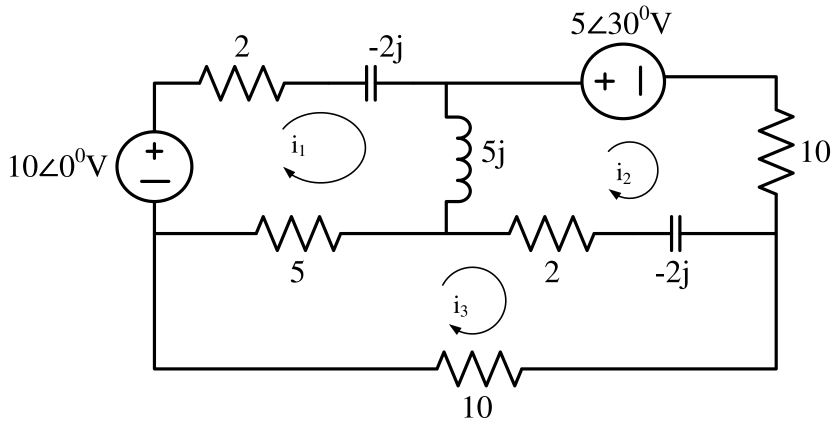

Figure: 33 Network with meshes

There are 3 meshes in the given network as 1, 2 and 3.

KVL to mesh 1

$-10\angle 0^0 + 2i_1 - 2ji_1 + 5j(i_1 - i_2) + 5(i_1 - i_3) = 0$

$2i_1 - 2ji_1 + 5ji_1 - 5ji_2 + 5i_1 - 5i_3 = 10$

$(7+3j)\, i_1 - 5ji_2 - 5i_3 = 10$ --(1)

KVL to mesh 2

$5\angle 30^0 + 10i_2 - 2j(i_2 - i_3) + 2(i_2 - i_3) + 5j(i_2 - i_1) = 0$

$10i_2 - 2ji_2 - 2ji_3 + 2i_2 - 2i_3 + 5ji_2 - 5ji_1 = -5\angle 30^0$

$-5ji_1 + (12+3j)\, i_2 - (2 - 2j)\, i_3 = -(4.3301 + 2.5j)$ -------------(2)

KVL to mesh 3

$10i_3 + 5(i_3 - i_1) + 2(i_3 - i_2) - 2j(i_3 - i_2) = 0$

$10i_3 + 5i_3 - 5i_1 + 2i_3 - 2i_2 - 2ji_3 - 2i_2 = 0$

$-5i_1 - (2 - 2j)\, i_2 + (17 - 2j)\, i_3 = 0$ --------------------------------(3)

Equations (1), (2) and (3) in matrix form

$$\begin{bmatrix} 7+3j & -5j & -5 \\ -5j & 12+3j & -(2-2j) \\ -5 & -(2-2j) & 17-2j \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -(4.3301+2.5j) \\ 0 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 7+3j & -5j & -5 \\ -5j & 12+3j & -(2-2j) \\ -5 & -(2-2j) & 17-2j \end{bmatrix} = 1390 + 650\mathrm{i}$$

$$\Delta_1 = \begin{bmatrix} 10 & -5j & -5 \\ -(4.3301+2.5j) & 12+3j & -(2-2j) \\ 0 & -(2-2j) & 17-2j \end{bmatrix} = 2200.9 - 24.757\mathrm{i}$$

$$\Delta_2 = \begin{bmatrix} 7+3j & 10 & -5 \\ -5j & -(4.3301+2.5j) & -(2-2j) \\ -5 & 0 & 17-2j \end{bmatrix} = -\ 140.51\ +\ 339.79\mathrm{i}$$

$$\Delta_3 = \begin{bmatrix} 7+3j & -5j & 10 \\ -5j & 12+3j & -(4.3301+2.5j) \\ -5 & -(2-2j) & 0 \end{bmatrix} = 655.90\ +\ 126.39\mathrm{i}$$

Where

$i_1 = \Delta 1/\Delta = 1.2924 - 0.6222iA = 1.4344 \angle - 25.71$

$i_2 = \Delta 2/\Delta = 0.0109 + 0.2394iA = 0.2396 \angle 87.39$

$i_3 = \Delta 3/\Delta = 0.4221 - 0.1065iA = 0.4353 \angle -14.16$

Q9. Find the voltage drop across resistance R in the network shown in Fig. using mesh analysis.

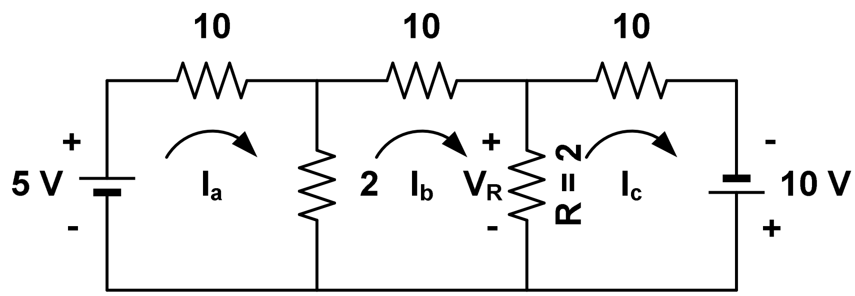

Figure: 34 Q9

Answer:

There are total 3 meshes in the given network

KVL to mesh 1

$10I_a + 2(I_a - I_b) -5 = 0$

$10I_a + 2I_a - 2I_b = 5$

$12I_a - 2I_b = 5$ ----------------------------(1)

KVL to mesh 2

$10I_b + 2(I_b - I_a) + 2(I_b - I_c) = 0$

$10I_b + 2I_b - 2I_a + 2I_b - 2I_c = 0$

$-2I_a + 14I_b -2I_c =0$ ---------------------(2)

KVL to mesh 3

$10I_c + 2(I_c - I_b) -10 = 0$

$10I_c + 2I_c - 2I_b = 10$

$- 2I_b + 12I_c = 10$ -----------------------(3)

Writing equations (1), (2) and (3) in matrix form as shown below

$$\begin{bmatrix} 12 & -2 & 0 \\ -2 & 14 & -2 \\ 0 & -2 & 12 \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 10 \end{bmatrix}$$

$\Delta = \begin{bmatrix} 12 & -2 & 0 \\ -2 & 14 & -2 \\ 0 & -2 & 12 \end{bmatrix}$

$\Delta = 12\big((14 \times 12) - (-2 \times -2)\big) - (-2)\big((12 \times -2) - (0 \times -2)\big) + 0$

$\Delta = 12(168 - 4) + 2(-24 - 0) + 0$

$\Delta = 12(164) + 2(-24)$

$\Delta = 1920$

$\Delta 1 = \begin{bmatrix} 5 & -2 & 0 \\ 0 & 14 & -2 \\ 10 & -2 & 12 \end{bmatrix} = 860$

$\Delta 2 = \begin{bmatrix} 12 & 5 & 0 \\ -2 & 0 & -2 \\ 0 & 10 & 12 \end{bmatrix} = 360$

$\Delta 3 = \begin{bmatrix} 12 & -2 & 0 \\ -2 & 14 & -2 \\ 0 & -2 & 12 \end{bmatrix} = 1660$

$I_a = \Delta 1/\Delta = 860/1920 = 0.45A$

$I_b = \Delta 2/\Delta = 360/1920 = 0.1875A$

$I_c = \Delta 3/\Delta = 1660/1920 = 0.8645A$

Q10. Use mesh analysis to determine mesh currents for the network shown in Fig.

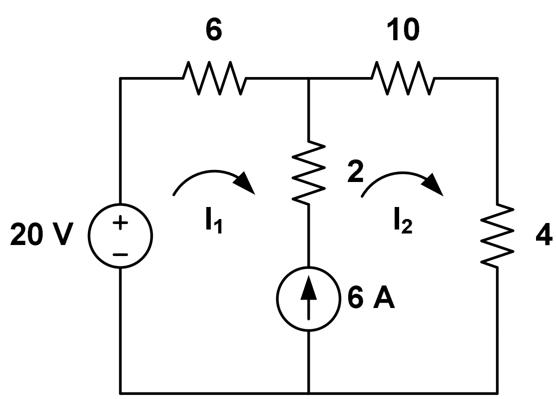

Figure: 35 Q10

Answer:

Apparently there are two meshes in the above network but as a independent current source of 6A is connected between two meshes so they are forming a supermesh.

KVL to the supermesh

$-20 + 6I_1 + 10I_2 + 4I_2 = 0$

$6I_1 + 14I_2 = 20$ -------------------(1)

Equation from supermesh

$-I_1 + I_2 = 6$-------------------------(2)

Equations (1) and (2) in matrix form

$$\begin{bmatrix} 6 & 14 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 6 \end{bmatrix}$$

Solving above matrix

$I_1 = -3.2A$

$I_2 = 2.8A$

References

1. Network Analysis, 3rd Edition M. E. Van Valkenburg PHI Learning Private Limited.
2. Sudhakar, A., Shyammohan, S. P.; "Circuits and Network"; Tata McGraw-Hill New Delhi, 1994.
3. A William Hayt, "Engineering Circuit Analysis" 8th Edition, McGraw-Hill Education.
4. C. K. Alexander and M. N. O. Sadiku, "Fundamentals of Electric Circuits" 5th Edition, McGraw-Hill Education.

UNIT No 3

UNIT-3: Network Theorem
Superposition Theorem, Thevenin's Theorem, Norton's Theorem, Maximum Power Transfer Theorem

1 SUPERPOSITION THEOREM

Q1.using superposition theorem, determine current I for the network shown in fig below.

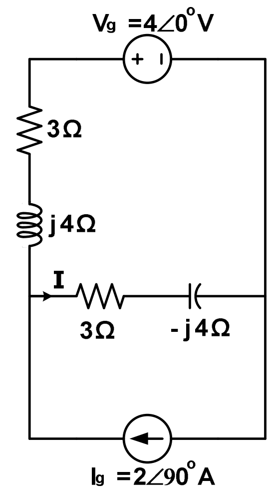

Figure: 36 Q1

Answer:

First consider only voltage source and make the current source deactivate i. e open circuit it as shown below

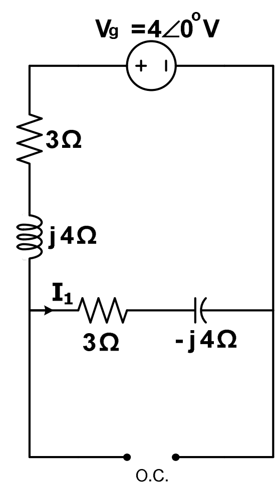

Figure: 37 Network with C.S. open

$I_1 = \frac{V}{R} = \frac{4\angle 0^0}{3+4j+3-4j} = \frac{4}{6} = 0.667A$

Next consider only current source and make the voltage source deactivate i. short circuit it as shown below

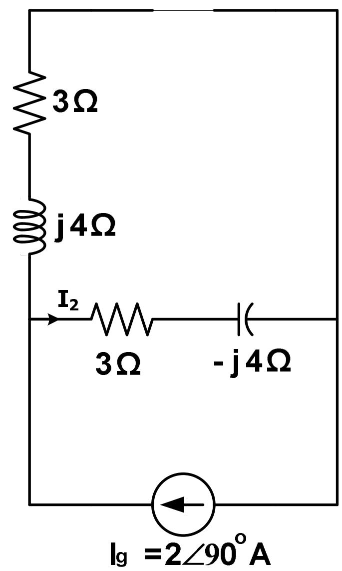

Figure: 38 Network with V.S.1 short

Using current division rule

$I_2 = \frac{3+4j}{3+4j+3-4j} \times 2\angle 90^0$

I_2 =-1.333+1j =(-1.333+j) A

Total current I = $I_1 + I_2$

= 0.667 – 1.333 – j

I = (-0.666 – j) A

Q2. Using superposition theorem, find the current i_4 for the network shown in below.

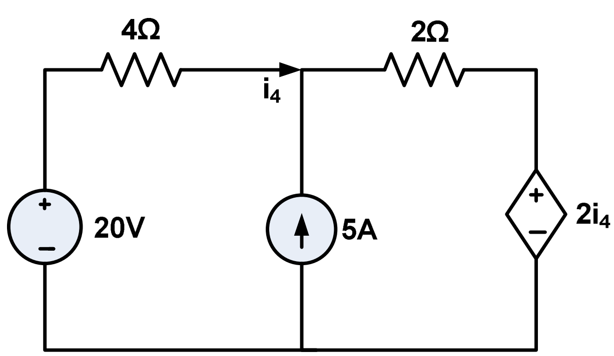

Figure: 39 Q2

Answer:

There are 3 energy sources in the given network, one independent current source, one independent voltage source and one dependent voltage source.
Superposition theorem does not have any effect on dependent energy sources.
First consider only voltage source and make the current source deactivate i. e open circuit it as shown below

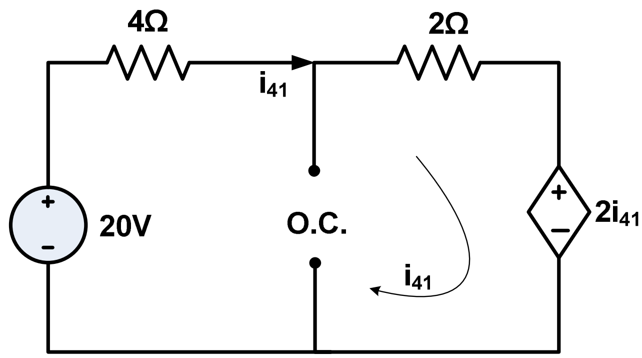

Figure: 40 Network with C.S. open

Above network is forming a mesh with mesh current i_4.
KCL to the mesh
$-20 + 4i_{41} + 2i_{41} + 2i_{41} = 0$
$8i_{41} = 20$
$i_{41} = 20/8 = 2.5A$
Next consider only current source and make the voltage source deactivate i. short circuit it as shown below

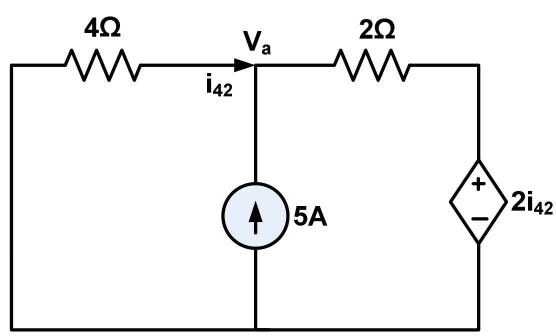

Figure: 41 Network with V.S. short

Consider the node a with nodal voltage V_a. KCL at node a

$$\frac{V_a}{4} + \frac{V_a - 2i_{42}}{2} = 5$$

Where

$$\frac{V_a}{4} + \frac{V_a}{2} - \frac{2i_{42}}{2} = 5$$

$$i_{42} = -\frac{V_a}{4}$$

Substituting in above equation

$$\frac{V_a}{4} + \frac{V_a}{2} + \frac{V_a}{4} = 5$$

$i_{42} = -1.25A$

So total current is

$i_4 = i_{41} + i_{42} = 1.25A$

Q3. Using superposition theorem, find the current i for the network shown in below. M

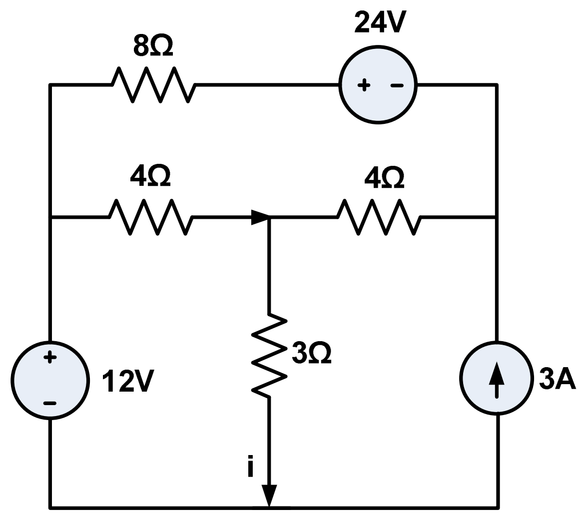

Figure: 42 Q3

There are 2 independent voltages sources and 1 current source.

First consider only current source and make the 2 voltage sources deactivate i.e. short circuit it as shown below

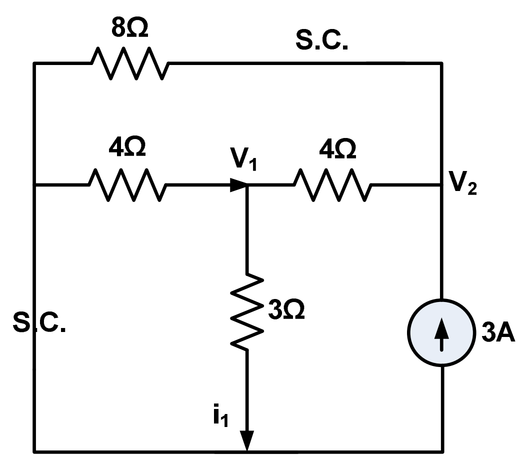

Figure: 43 Network with V.S. short

KCL at node 1 with nodal voltage V_1

$$\frac{V_1}{4} + \frac{V_1}{3} + \frac{V_1 - V_2}{4} = 0$$

$$\frac{V_1}{4} + \frac{V_1}{3} + \frac{V_1}{4} - \frac{V_2}{4} = 0$$

$$V_1\left[\frac{1}{4}+\frac{1}{3}+\frac{1}{4}\right]+V_2\left[-\frac{1}{4}\right]=0$$

$V_1[0.833]+V_2[-0.25]=0$-----------------------(1)

KCL at node 2 with nodal voltage V_2

$$\frac{V_2}{8}+\frac{V_2-V_1}{4}=3$$

$$\frac{V_2}{8}+\frac{V_2}{4}-\frac{V_1}{4}=3$$

$V_1[-0.25]+V_2[0.375]=3$-----------------------(2)

Writing equations (1) and (2) in matrix form

$$\begin{bmatrix}0.833 & -0.25\\ -0.25 & 0.375\end{bmatrix}\begin{bmatrix}V_1\\ V_2\end{bmatrix}=\begin{bmatrix}0\\ 3\end{bmatrix}$$

Solving above matrix

$V_1 = 3V$, and $V_2 = 10V$

Then $i_1 = V_1/R = 3/3 = 1A$

Next consider only 1 voltage source of 12 volts and deactivate the another voltage source and current source i.e. short circuit the voltage source and current source is to be open circuit as shown below

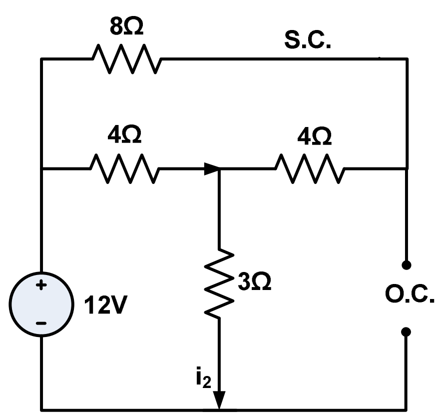

Figure: 44 Network with C.S. open

In the above figure 8Ω and 4Ω resistors are in series so adding them and redrawn the network

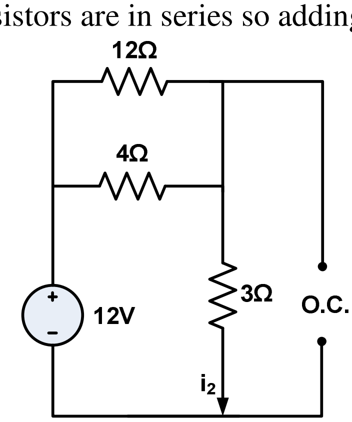

Figure: 45 Network with C.S. open

Now in the above figure 12Ω and 4Ω resistors are in parallel so simplifying and redrawn the network

12Ω and 4Ω resistors are in parallel =

$$R_{Pal} = \frac{12 \times 4}{12 + 4} = 3\Omega$$

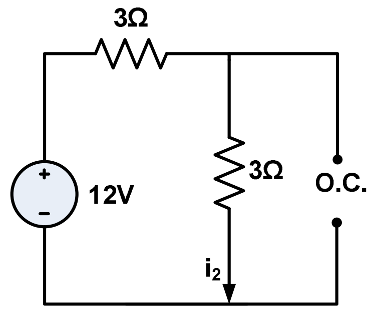

Figure: 46 Network with C.S. open

$$i_2 = \frac{V}{R} = \frac{12}{3+3} = \frac{12}{6} = 2A$$

Next consider only 1 voltage source of 24 volts and deactivate the another voltage source and current source i.e. short circuit the voltage source and current source is to be open circuit as shown below

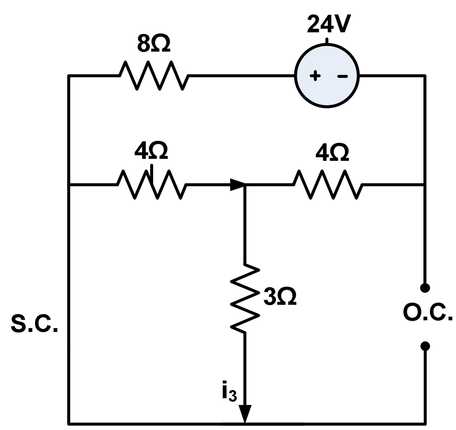

Figure: 47 Network with C.S. open

There are 2 meshes in the above network so consider 2 mesh currents as i_2 and i_3 as shown below

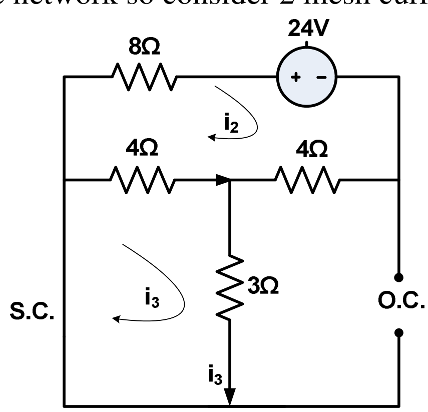

Figure: 48 Network with C.S. open

KVL to mesh 1 with current i_2

$8i_2 + 24 + 4i_2 + 4(i_2 - i_3) = 0$

$8i_2 + 4i_2 + 4i_2 – 4i_3 = -24$
$16i_2 – 4i_3 = -24$ ----------------------(1)

KVL to mesh 2 with current i_3
$3i_3 + 4\,(i_3 – i_2) = 0$
$3i_3 + 4i_3 – 4i_2 = 0$
$7i_3 – 4i_2 = 0$
$– 4i_2 + 7i_3 = 0$ ------------------------(2)

Equations (1) and (2) in matrix form

$$\begin{bmatrix} 16 & -4 \\ -4 & 7 \end{bmatrix}\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -24 \\ 0 \end{bmatrix}$$

Solving above matrix
$i_2 = -1.75A$ and $i_3 = -1A$
Total $i = i_1 + i_2 + i_3 = 1 + 2 + (-1) = 2A$

Q4. Using Thevenin's theorem, evaluate the current I in the impedance Z = 2 + 3j connected between nodes 'a' and 'b' of Figure shown below.

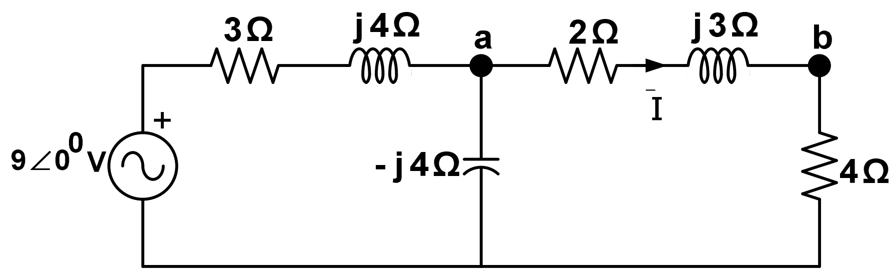

Figure: 49 Q4

Answer:

Determination of V_{th} i.e. thevenin's voltage remove the impedance across 'a' and 'b' as shown below

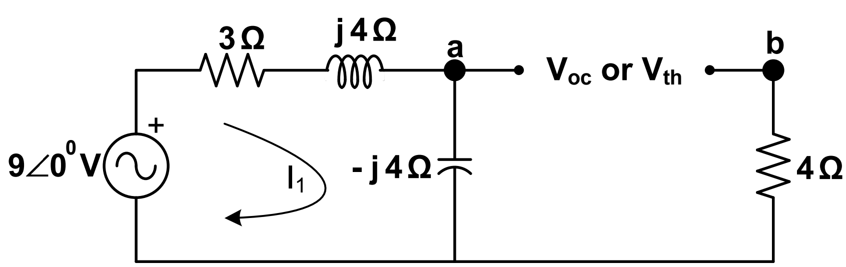

Figure: 50 Network with V_{th}

KVL to loop 1 with current I_1
$-9 + (3 + 4j)\, I_1 –j4I_1 = 0$
$3I_1 + 4jI_1 –j4I_1 = 9$

$3I_1 = 9$

$I_1 = 9/3 = 3A$

Thevenin's voltage = $V_{oc} = V_{th} = V_a - V_b = V_a$

As V_b is 0.

$V_{oc} = V_{th} = V_a = IR = (-j4)(3) = -12jV = 12\angle -90^0$ V

Determination of Z_{th} i.e. thevenin's impedance short the voltage source as shown below

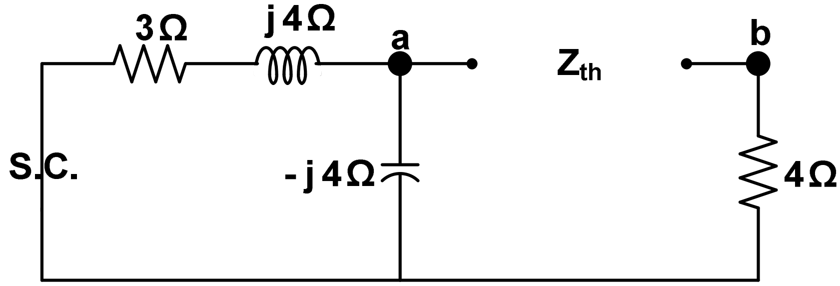

Figure: 51 Network for Z_{th}

Zth = (3 + 4j) || (-j4) + 4

$$Z_{th} = \frac{(3+4j)\times(-j4)}{(3+4j)+(-4j)} + 4 = 9.33 - 4j = 10.125\angle - 23.2^0\Omega$$

Thevenin's equivalent circuit is as follows

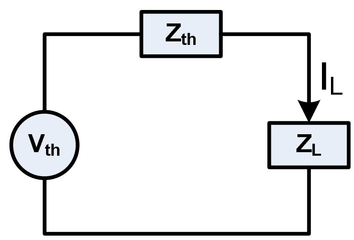

Figure: 52 Network with V_{th} and Z_{th}

Where $V_{th} = -12jV = 12\angle -90^0$ V

$Z_{th} = 9.33 - 4j = 10.125\angle - 23.2^0\Omega$

$Z_L = 2 + 3j$

$$I_L = \frac{V}{R} = \frac{12\angle -90^0}{9.33-4j+(2+3j)} = 0.092 - 1.05j = 1.06\angle - 85A$$

Q5. Find the Thevenin's equivalent between terminals a and b for the network shown below.

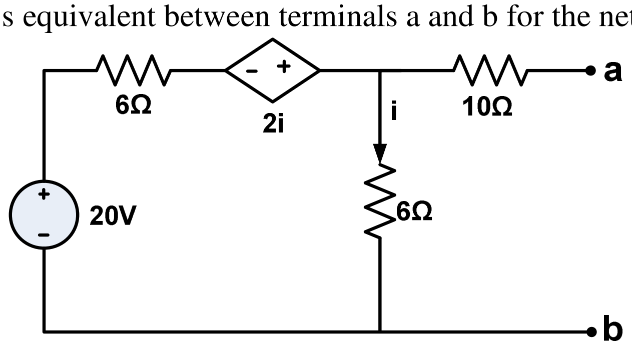

Figure: 53 Q5

Answer:

To find Thevenin's voltage V_{oc} or V_{th} as follows

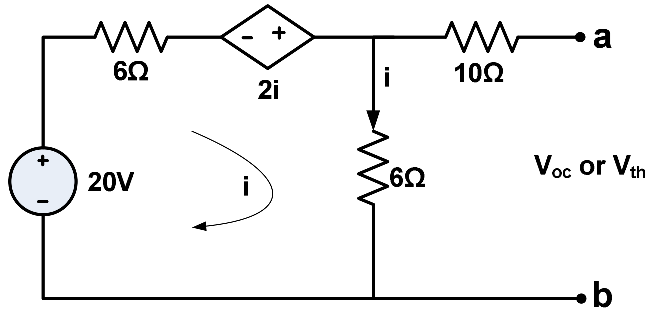

Figure: 54 Network for V_{th}

KVL to mesh 1 with current i

$-20 + 6i - 2i + 6i = 0$

$10i = 20$

$i = 20/10$

$i = 2A$

$V_{oc} = V_{th} = IR = 2.6 = 12V$

$V_{oc} = V_{th} = 12V$

To find R_{th} thevenin's resistor short the terminals a and b as follows

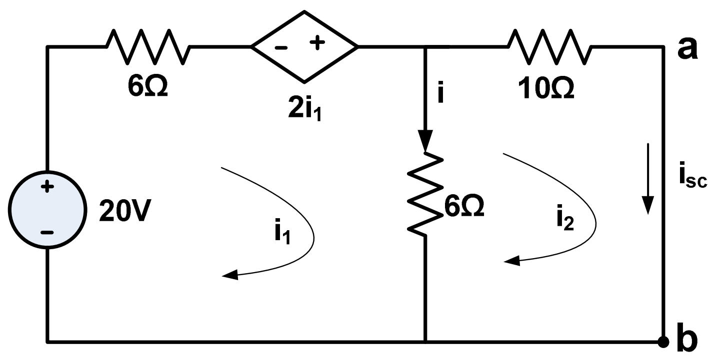

Figure: 55 Network with I_{sc}

KVL to mesh 1 with current i_1

$-20 + 6i_1 - 2i_1 + 6(i_1 - i_2) = 0$

$6i_1 - 2i_1 + 6i_1 - 6i_2 = 20$

$10i_1 - 6i_2 = 20$--------------------(1)

KVL to mesh 2 with current i_2

$10i_2 + 6(i_2 - i_1) = 0$

$10i_2 + 6i_2 - 6i_1 = 0$

$-6i_1 + 16i_2 = 0$--------------------(2)

Solving equation (1) and (2)
$i_1 = 2.58A$
$i_2 = 0.97A = i_{sc}$
$R_{th} = V_{oc} / i_{sc} = 13.6\Omega$

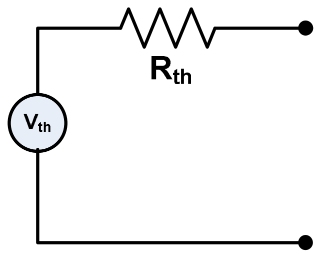

Figure: 56 Network with V_{th} and R_{th}

Q6. Find the Norton's equivalent between terminals a and b for the network shown below.

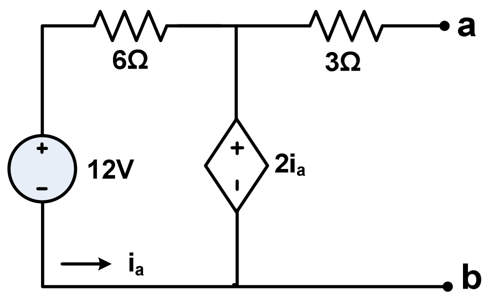

Figure: 57 Q6

Answer:
To find Norton's current I_N or I_{sc} i.e. short circuit current

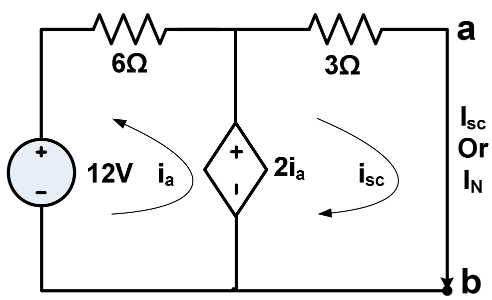

Figure: 58 Network for I_{sc}

KVL to mesh 1 with current i_a
$6i_a - 2i_a + 12 = 0$
$4i_a = -12$
$i_a = -12/4 = -3A$
KVL to mesh 2 with current i_{sc}
$-2i_a + 3i_{sc} = 0$
Substituting $I_a = -3$
$-2(-3) + 3i_{sc} = 0$
$6 + 3i_{sc} = 0$

$6 = -3i_{sc}$
$i_{sc} = 6/-3 = -2A$

To find R_N consider the given network

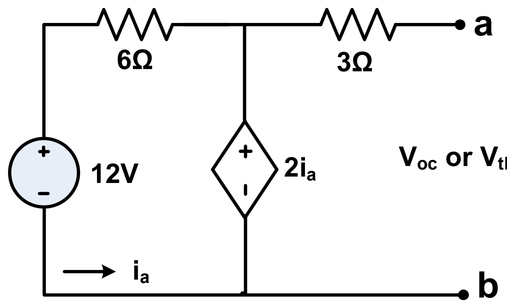

Figure: 59 Network for V_{th}

Where $i_a = -3A$
$V_{oc} = IR = 2i_a = 2(-3) = -6V$
$$R_N = \frac{V_{oc}}{i_{sc}} = \frac{-6}{-2} = 3\Omega$$
$R_N = 3\Omega$
Norton's equivalent circuit as

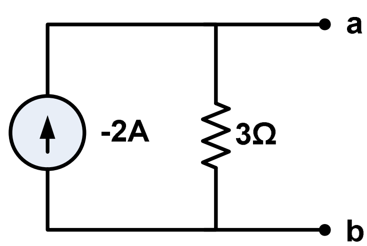

Figure: 60 Network with I_N and R_N

Q7. Find the current through R_L using Norton's theorem for the network shown below.

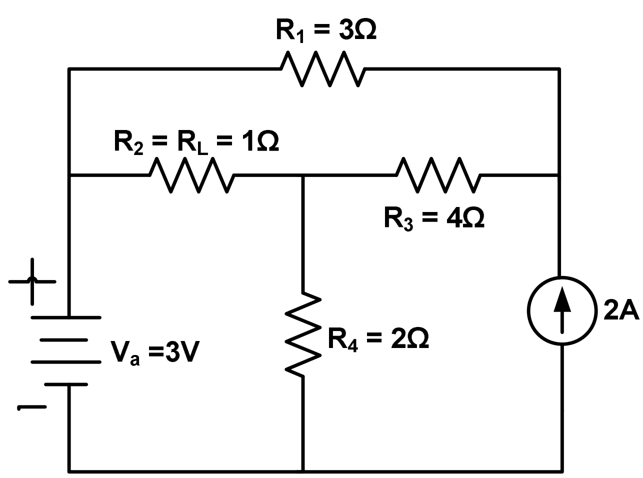

Figure: 61 Q7

Answer:

Remove the R_L through which the Current is to be found out and short the terminals as shown below

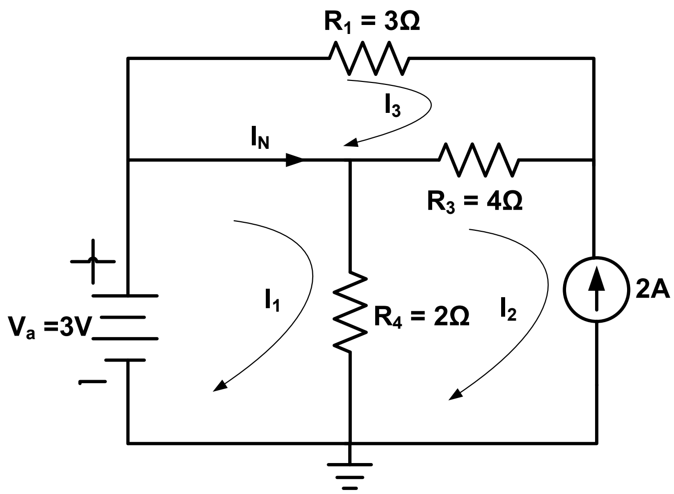

Figure: 62 Network with I_N

To find I_N
There are 2 meshes in the above network with currents I_N and I_2
KVL to loop 1 with current I_1
$-3 + 2(I_1 – I_2) = 0$
$2I_1 – 2I_2 = 3$----------------------(1)

KVL to loop 3 with current I3
$3I_3 + 4(I_3 – I_2) = 0$
$3I_3 + 4I_3 – 4I_2 = 0$
$7I_3 – 4I_2 = 0$ --------------------(2)

As an independent current source of 2A is directly connected to mesh 2 with current I_2 but the direction of I_2 is exactly opposite to the connected current source
So $I_2 = -2A$
Substituting in equation (1)
$2I_1 – 2(-2) = 3$----------------------(1)

$2I_1 + 4 = 3$
$2I_1 = 3 - 4$
$2I_1 = -1$
$I_1 = -0.5A$
Substituting in equation (2)
$7I_3 – 4(-2) = 0$ --------------------(2)

$7I_3 + 8 = 0$
$7I_3 = -8$
$I_3 = -1.1428A$

From figure
$I_N = I_1 - I_3 = -2 = -0.5 - (-1.14) = 0.64$ A

To find Norton's resistance i.e. R_N short the voltage source and open the current source as shown below

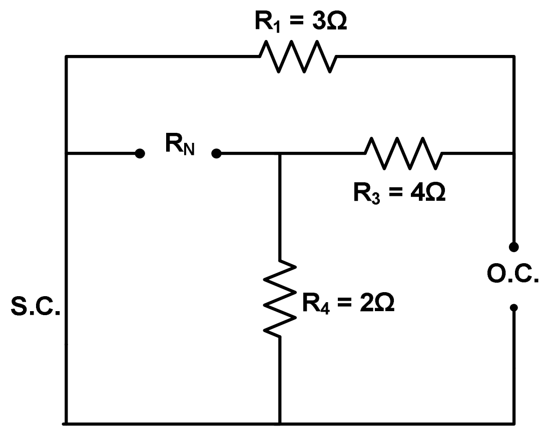

Figure: 63 Network with C.S. open

$R_N = (R_1 + R_3) \parallel R_4$
$R_1 + R_3 = 3 + 4 = 7$
$R_N = 7\|2 = (7.2)/ 7 + 2 = 1.555\Omega$
$R_N = 1.555\Omega$

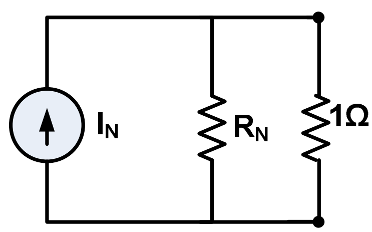

Figure: 64 Network with I_N and R_N

Q8. Obtain the Norton's equivalent of the network shown in Fig., at terminals xy.

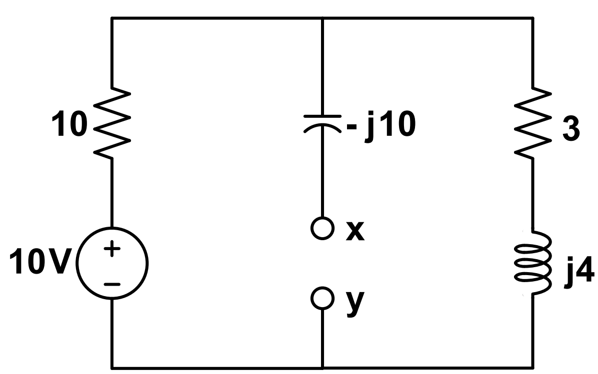

Figure: 65 Q8

Answer:

To find I_N short the x and y terminals as below

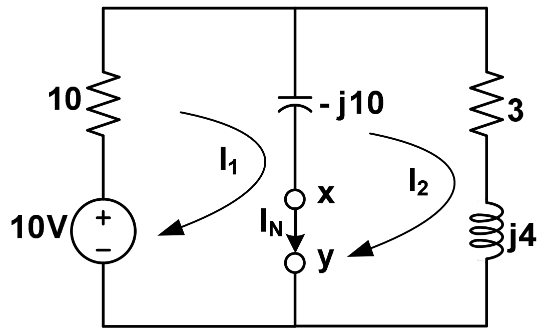

Figure: 66 Network with I_N

As seen from above network there are 2 meshes with current I_1 and I_2.
KVL to mesh 1 with current I_1
$-10 + 10I_1 - j10(I_1 - I_2) = 0$
$10I_1 - j10I_1 + j10I_2 = 10$
$(10 - j10)I_1 + j10I_2 = 10$----------------(1)

KVL to mesh 2 with current I_2
$-j10(I_2 - I_1) + 3i_2 + j4I_2 = 0$
$-j10I_2 + j10I_1 + 3i_2 + j4I_2 = 0$
$j10I_1 + (3 - j6)I_2 = 0$ -----------------------(2)
equations (1) and (2) in matrix form

$$\begin{bmatrix} 10 - j10 & j10 \\ j10 & 3 - j6 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

Solving above matrix
$I_1 = (0.58 - 0.12i)$A and $I_2 = (0.69 - 0.54i)$A
$I_N = I_1 - I_2 = 0.4404\angle 73.87^0 = (-0.1223 - 0.423i)$A

To determine Z_N i.e. Norton's impedance short the voltage source as shown below

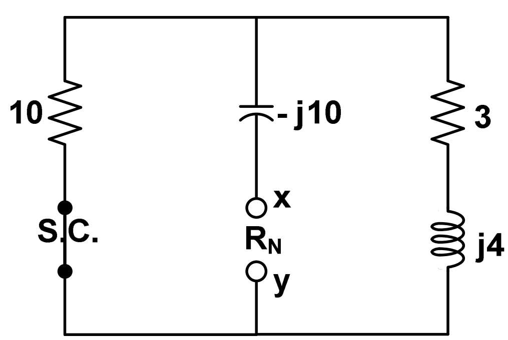

Figure: 67 Network for R_N

$Z_N = 10 \parallel (3 + j4) + (-j10) = 2.9729 - 7.8378i\ \Omega = 8.38\ \angle{-69.22}\Omega$

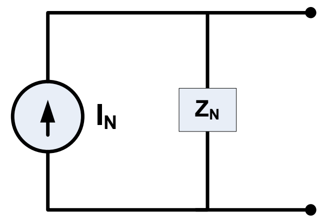

Figure: 68 Network with I_N and Z_N

Q9. Find the load resistance R_L that enables the circuit between terminals a and b to deliver maximum power to the load. Find the maximum power delivered to the load.

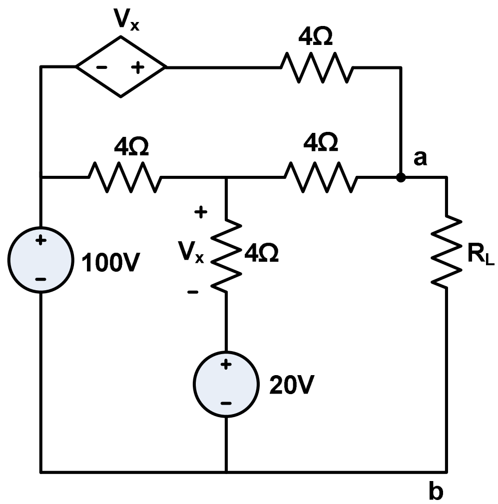

Figure: 69 Q9

Answer:
To find thevenin's voltage i.e. V_{th} or V_{oc} i.e. open circuit voltage between terminals a and b remove RL i.e. Load resistor as shown below

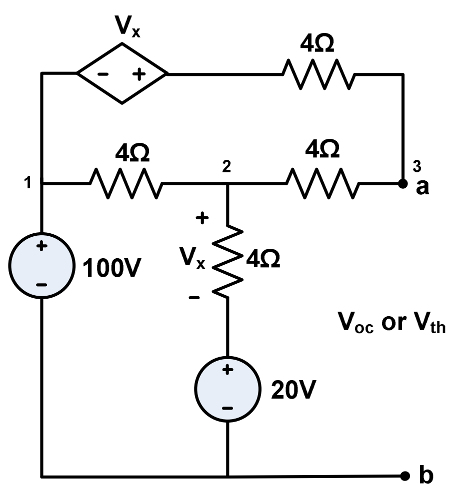

Figure: 70 Network with V_{th}

KCL at node 1 is not needed as V_1 i.e. voltage at node 1 is 100V directly from above network.

KCL at node 2

$$\frac{V_2 - 100}{4} + \frac{V_2 - 20}{4} + \frac{V_2 - V_3}{4} = 0$$
$$\frac{V_2}{4} - \frac{100}{4} + \frac{V_2}{4} - \frac{20}{4} + \frac{V_2}{4} - \frac{V_3}{4} = 0$$
$$3V_2 - V_3 = 120 - - - - - (1)$$

KCL at node 3

$$\frac{V_3 - V_x - 100}{4} + \frac{V_3 - V_2}{4} = 0$$
$$V_x = V_2 - 20$$
$$\frac{V_3 - (V_2 - 20) - 100}{4} + \frac{V_3 - V_2}{4} = 0$$
$$\frac{V_3}{4} - \frac{V_2}{4} + \frac{20}{4} - \frac{100}{4} + \frac{V_3}{4} - \frac{V_2}{4} = 0$$
$$-\frac{V_2}{4} - \frac{V_2}{4} + \frac{V_3}{4} + \frac{V_3}{4} = -\frac{20}{4} + \frac{100}{4}$$
$$V_2\left(-\frac{1}{4} - \frac{1}{4}\right) + V_3\left(\frac{1}{4} + \frac{1}{4}\right) = 20$$
$$V_2(-0.5) + V_3(0.5) = 20\text{--------}(2)$$

Solving equation (1) and (2)
$V_{th} = V_{oc} = V_a – V_b$
But $V_b = 0$
So $V_{th} = V_{oc} = V_a = 120V$
To find R_{th} apply a 1V voltage source and deactivate 100V voltage source as shown below

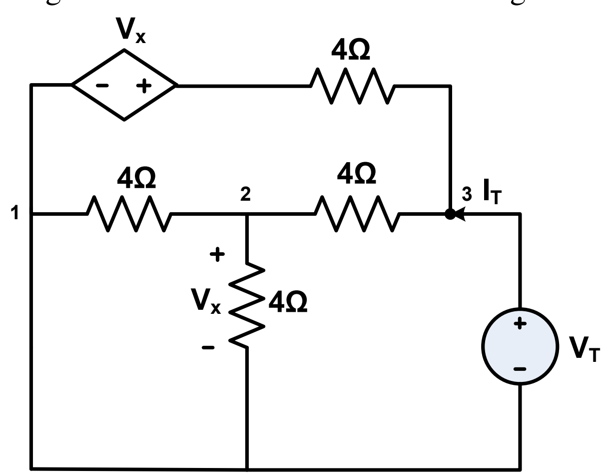

Figure: 71 Network with V_T

From above figure $V_2 = V_x$ and $V_3 = V_T$

KCL at node 2

$$\frac{V_2}{4}+\frac{V_2}{4}+\frac{V_2-V_3}{4}=0$$
$$\frac{V_2}{4}+\frac{V_2}{4}+\frac{V_2}{4}-\frac{V_3}{4}=0$$
$$V_2\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)-V_3\left(\frac{1}{4}\right)=0$$

$V_2(0.75) – V_3(0.25) = 0$

$0.75V_2 – 0.25V_3 = 0$ -------------(1)

KCL at node 3

$$\frac{V_3-V_2}{4}+\frac{V_3-V_x}{4}-I_T=0$$
$$\frac{V_3}{4}-\frac{V_2}{4}+\frac{V_3}{4}-\frac{V_x}{4}-I_T=0$$
$$V_3\left(\frac{1}{4}+\frac{1}{4}\right)+V_2\left(-\frac{1}{4}\right)+V_x\left(-\frac{1}{4}\right)=I_T$$

$V_3 (0.5) + V_2(-0.25) + V_X(-0.25) = I_T$

Where $V_2 = V_x$

$V_3 (0.5) + V_2(-0.25) + V_2(-0.25) = I_T$

$0.5V_3 - 0.5V_2 = I_T$ -----------------(2)

From equations (1) and (2)

$$\frac{V_T}{I_T}=3.03\Omega=R_{th}=R_L$$

Thevenin's equivalent circuit is as follows

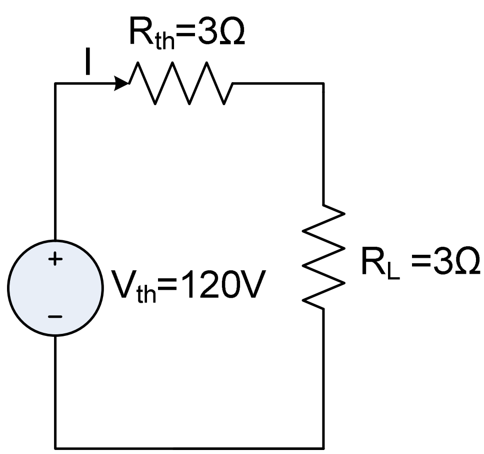

Figure: 72 Network with V_{th} and R_{th}

To get maximum power transfer
$R_{th} = R_L = 3\Omega$
So
$I^2R_L=(120/6)^2\times 3=1200W$

Q10. What should be the value of Z_L in Fig., connected across 'a' and 'b', so that it will draw maximum power. What is the amount of this maximum power?

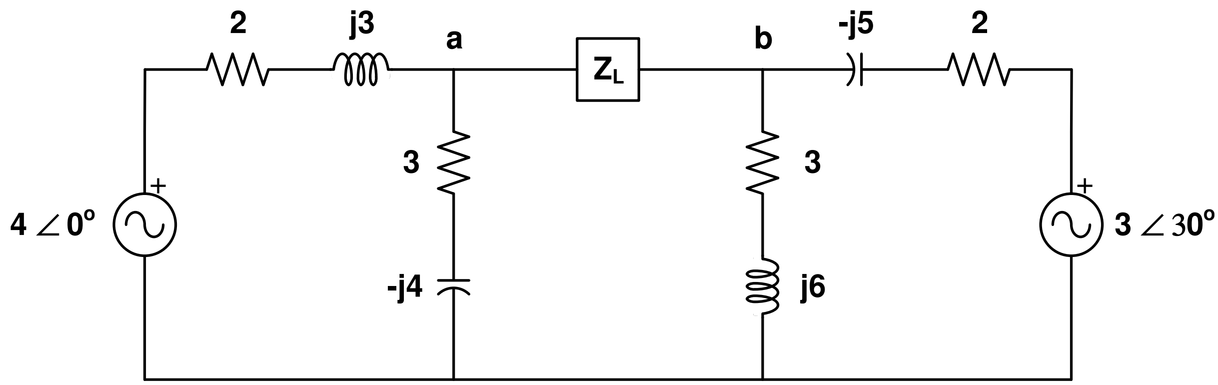

Figure: 73 Q10

Answer:

To find Thevenin's voltage i.e. V_{th} or V_{oc} as shown below

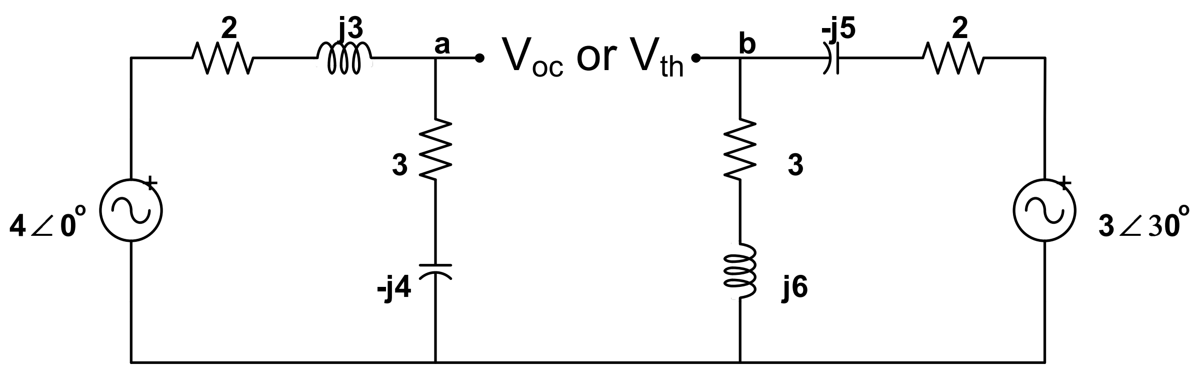

Figure: 74 Network with V_{th}

$V_{ab} = V_a - V_b = V_{th} = V_{oc}$
To find V_a
Using voltage divider rule

$$V_a = 4\angle 0 \frac{(3-4j)}{(2+3j+3-j4)}$$

To find V_b
Using voltage divider rule

$$V_b = 3\angle 30^0 \frac{(3+6j)}{(2+3j+3+j6)}$$

$V_{ab} = V_a - V_b = V_{th} = V_{oc} = 1.2 - 0.04j = 1.2\angle -1.92^0 V$

To find R_{th} short both the voltage sources as shown below

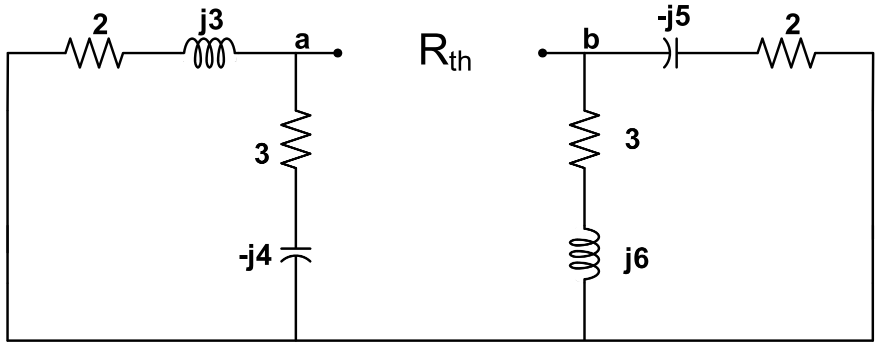

Figure: 75 Network for R_{th}

R_{th} = (2 + 3j) || (3 – 4j) + (2 – 5j) || (3 + 6j) = 7.12 + 1.08j Ω

Maximum power drawn

$$P_{Lmax} = \frac{V_{th}^2}{4 \times R_{th}} = 0.0506Watts$$

References

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2. Sudhakar, A., Shyammohan, S. P.; “Circuits and Network”; Tata McGraw-Hill New Delhi, 1994.
3. A William Hayt, “Engineering Circuit Analysis” 8th Edition, McGraw-Hill Education.
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UNIT No 4

UNIT-4: Initial and Final Conditions, Impedance Functions and Circuit Analysis with Laplace Transform

Concept of initial and final conditions, behavior of resistor, inductor and capacitor at t = 0- and at t = 0+, procedure for evaluating initial and final conditions, analytical treatment. Review of Laplace Transform, concept of complex frequency, transform impedance and admittance, s – domain impedance and admittance models for resistor, inductor and capacitor, series and parallel combinations of elements. Transformed network on loop and mesh basis, mesh and node equations for transformed networks, time response of electrical network with and without initial conditions by Laplace transform.

1 INITIAL AND FINAL CONDITIONS

Q1. In the given network, K is closed at t= 0 with zero current in the inductor. Find the values of i, di/dt and d^2i/dt^2 at t = 0+ if R = 10 Ω, L = 1 H and V = 100 V.

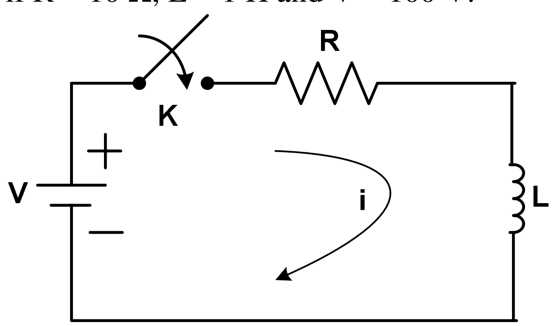


Figure: 76 Q1

Answer:

Step I: Network at t = 0-

Given that the current in the inductor is zero at the instant of switching.

So $I_L(0-) = 0$

Step II: Network at t = 0+

As switch K is closed at t= 0 so it will be closed at t = 0+

As the inductor opposes sudden change in its current

Then $I_L(0-) = I_L(0+) = 0$ so that inductor will be replaced by open circuit

Network at t = 0+ is as shown below

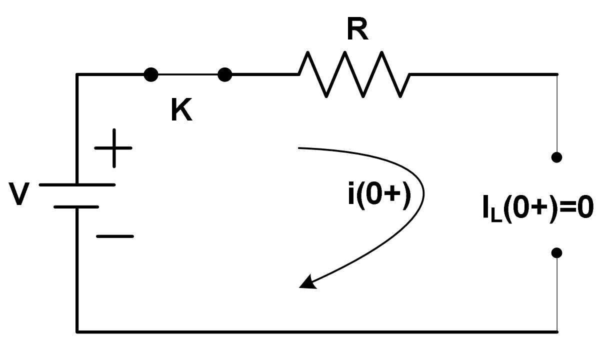


Figure: 77 Network at t = 0+

From the above network
i(0+) = 0

Step III: Network at t > 0

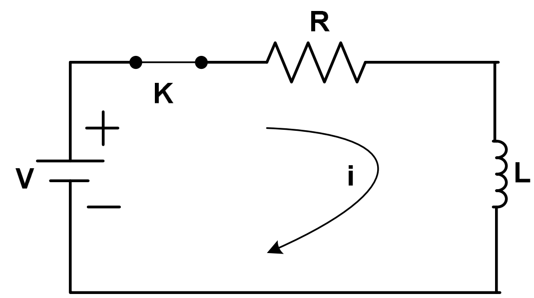


Figure: 76 Network at t > 0

KVL to above mesh with current i

$$R\frac{di}{dt}+L\frac{di}{dt}=V-----(1)$$

At t = 0+ equation (1) becomes

$$Ri(0+)+L\frac{di}{dt}(0+)=V$$

$$\frac{di}{dt}(0+)=\frac{V-Ri(0+)}{L}=\frac{100-10\times 0}{1}=\frac{100A}{sec}$$

Differentiating equation (1) with respect to time t as

$$R\frac{di}{dt}+L\frac{d^2i}{dt^2}=\frac{dV}{dt}=0-----(2)$$

At t = 0+ equation (2) becomes

$$R\frac{di}{dt}(0+)+L\frac{d^2i}{dt^2}(0+)=0$$

$$\frac{d^2i}{dt^2}(0+)=-\frac{Rdi}{Ldt}(0+)=-\frac{10}{1}\times 100=-1000A/sec^2$$

Q2. In the network of the Fig., the switch K is closed at t = 0 with the capacitor uncharged. Find values for i, di/dt and d^2i/dt^2 at t = 0+, for element values as follows: V = 100 V, R = 1000 Ω and C = 1 μF.

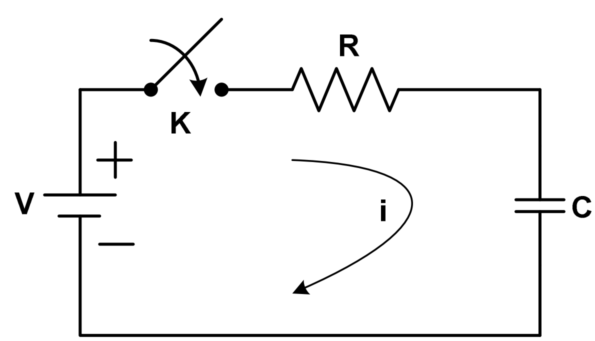


Figure: 78 Q2

Answer:

Step I: Network at t = 0-
Given that the capacitor is uncharged at the instant of switching so no voltage across it.
So $V_c(0-) = 0$

Step II: Network at t = 0+
As switch K is closed at t= 0 so it will remain closed at t = 0+
As the Capacitor opposes sudden change in its voltage
Then $V_c(0-) = V_c(0+) = 0$ so that capacitor will be replaced by short circuit
Network at t = 0+ is as shown below

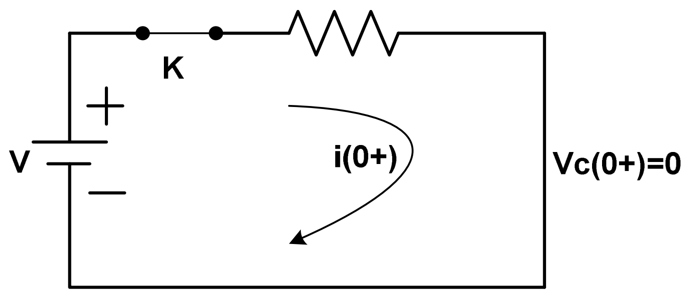


Figure: 79 Network at t = 0+

From the above network
$V_C(0+) = V_C(0-) = 0$
$$i(0+) = \frac{V}{R} = \frac{100}{1000} = 0.1A$$

Step III: Network at t > 0

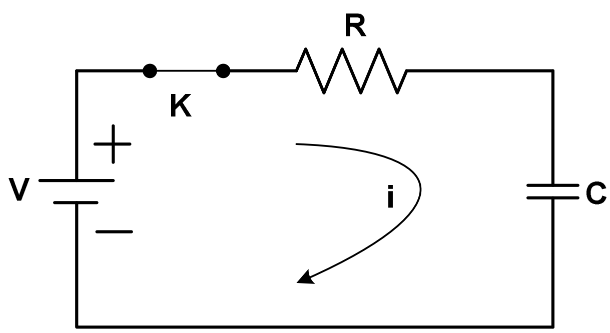


Figure: 80 Network at t > 0

KVL to above mesh with current i
$$Ri + \frac{1}{C}\int i dt = V ----(1)$$
Differentiating equation (1) with respect to time t as
$$R\frac{di}{dt} + \frac{i}{C} = \frac{dV}{dt} = 0 ------(2)$$
At t = 0+ equation (2) becomes
$$R\frac{di}{dt}(0+) + \frac{i(0+)}{C} = 0$$
$$\frac{di}{dt}(0+) = -\frac{i(0+)}{RC} = -\frac{0.1}{1000 \times 1 \times 10^{-6}} = -100A/sec$$

Differentiating equation (2) with respect to time t as

$$R\frac{d^2i}{dt^2}+\frac{i}{C}\frac{di}{dt}=0------(3)$$

At t = 0+ equation (3) becomes

$$\frac{d^2i}{dt^2}(0+)=-\frac{1}{RC}\frac{di}{dt}(0+)=-\frac{1}{1000\times 1\times 10^{-6}}\times(-100)=100000A/sec^2$$

Q3.In the network of the Fig., K is changed from position a to b at t = 0. Solve for i, di/dt and d^2i/dt^2 at t = 0+ if R = 1000 Ω, L = 1 H, C = 0.1 μF and V = 100 V.

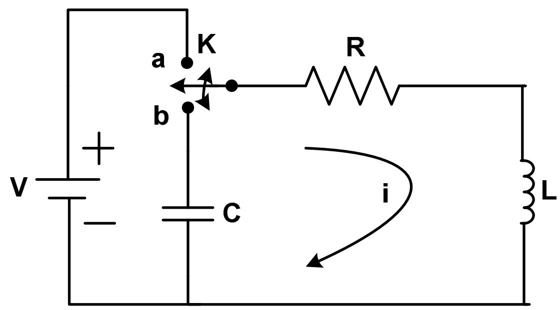


Figure: 81 Q3

Answer:

Step I: Network at t=0-

As switch k changes from position a to b at t = 0, it will be at a for t = 0-.

No condition is given for C and L so C is replaced by a Short circuit and L by Open circuit

Network at t=0- is as shown below

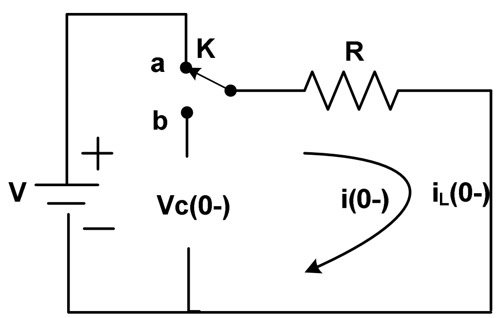


Figure: 82 Network at t = 0-

From the above network

$V_C(0-) = 0$

$$i(0-)=i_L(0-)=\frac{V}{R}=\frac{100}{1000}=0.1A$$

Step II: Network at t=0+

As switch k changes from position a to b at t = 0, it will be at b for t = 0+.

As the inductor opposes sudden change in its current

Then $I_L(0-) = I_L(0+) = 0.1A$ so that inductor will be replaced by current source of 0.1A.

As the Capacitor opposes sudden change in its voltage

Then $V_c(0-) = V_c(0+) = 0$ so that capacitor will be replaced by short circuit

Network at t = 0+ is as shown below

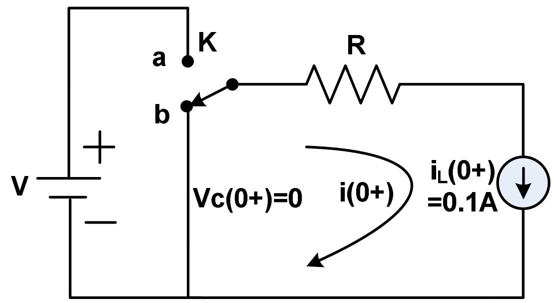


Figure: 83 Network at t = 0+

From above figure
$I_L(0-) = I_L(0+) = 0.1A$
$i(0+) = 0.1A$
Step III: Network at t > 0

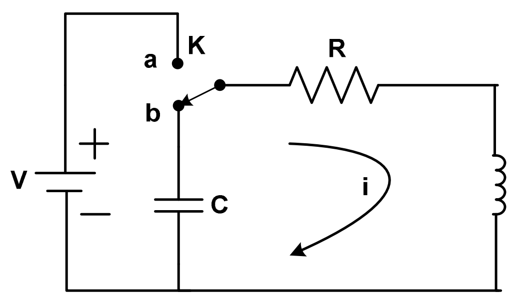


Figure: 84 Network at t > 0

KVL to above mesh with current i

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int idt = 0 - - - - (1)$$

Where

$$\frac{1}{C}\int idt = V_c(t)$$

At t = 0+ equation (1) will be

$$Ri(0+) + L\frac{di}{dt}(0+) + V_c(t) = 0$$

Substituting $I_L(0+) = 0.1A$ and $V_c(0+) = 0$ gives

$$\frac{di}{dt}(0+) = -\frac{Ri(0+)}{L} = -\frac{1000 \times 0.1}{1} = -100A/sec$$

Differentiating equation (1) with respect to time t as

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0 - - - - - (2)$$

At t = 0+ equation (2) becomes

$$R\frac{di}{dt}(0+) + L\frac{d^2i}{dt^2}(0+) + \frac{i(0+)}{C} = 0$$

$$\frac{d^2 i}{dt^2}(0+) = -\left[\frac{R\frac{di}{dt}(0+) + \frac{i(0+)}{C}}{L}\right] = -\left[\frac{1000 \times (-100) + \frac{0.1}{0.1 \times 10^{-6}}}{1}\right]$$
$$= -9 \times 10^5 A/sec^2$$

Q4. In the network given, the initial voltage on C_1 is V_1 and on C_2 is V_2 such that $v_1(0) = V_1$ and $v_2(0) = V_2$. At t = 0, the switch K is closed. Determine the values of dv_1/dt and dv_2/dt at t = 0+.

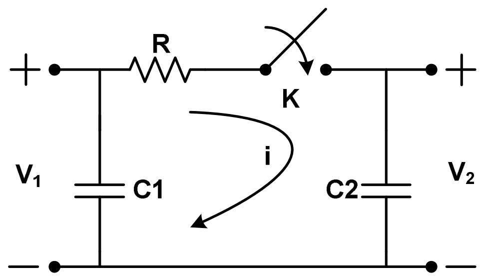


Figure: 85 Q4

Answer:
Step I: Network at t=0-
Given that the initial voltage on C_1 is V_1 and on C_2 is V_2 such that $v_1(0) = V_1$ and $v_2(0) = V_2$.
Therefore
$v_1(0-) = V_{c1}(0-) = V_1$ and
$v_2(0+) = V_{c1}(0+) = V_2$.

Step II: Network at t = 0+
As switch K is closed at t= 0 so it will remain closed at t = 0+
As the Capacitor opposes sudden change in its voltage
Then $V_{c1}(0-) = V_{c1}(0+) = V_1$ and Then $V_{c2}(0-) = V_{c2}(0+) = V_1$ therefore capacitor C_1 and C_2 will be replaced by voltage sources with voltages V_1 and V_2 respectively
Network at t = 0+ is as shown below

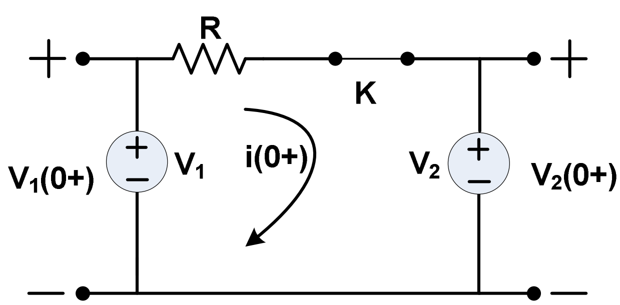


Figure: 86 Network at t = 0+

KVL to above loop
$V_1 - Ri(0+) - V_2 = 0$
$$i(0+) = \frac{V_1 - V_2}{R}$$
Step III: Network at t > 0

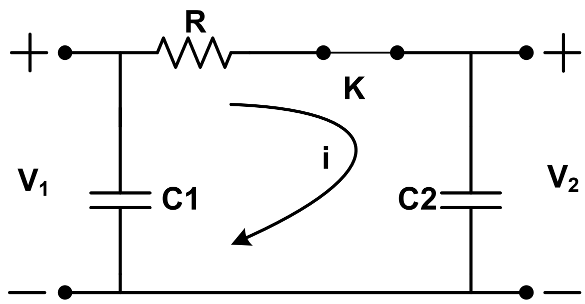


Figure: 87 Network at t > 0

From above network

$$V_1 = -\frac{1}{C_1}\int idt$$

$$V_2 = \frac{1}{C_2}\int idt$$

$$\frac{dv_1}{dt} = -\frac{i}{C_1} - - - - -(1)$$

$$\frac{dv_2}{dt} = -\frac{i}{C_2} - - - - -(2)$$

At t = 0+ equation (1) and (2) becomes

$$\frac{dv_1}{dt}(0+) = -\frac{i(0+)}{C_1} = \frac{\frac{V_1 - V_2}{RC_1}V}{sec}$$

$$\frac{dv_2}{dt}(0+) = \frac{i(0+)}{C_2} = \frac{V_1 - V_2}{RC_2}V/sec$$

Q5. The network shown in the accompanying Fig. is in the steady state with the switch K closed. At t = 0, the switch is opened. Determine the voltage across the switch, V_K and dV_K/dt at t = 0+.

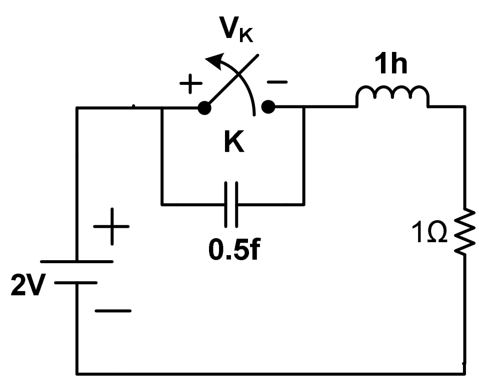


Figure: 88 Q5

Answer:

Step I: Network at t = 0-

As switch K is opened at t= 0 so it will remain closed at t = 0-

Inductor will be replaced by short circuit at t = 0- as shown below

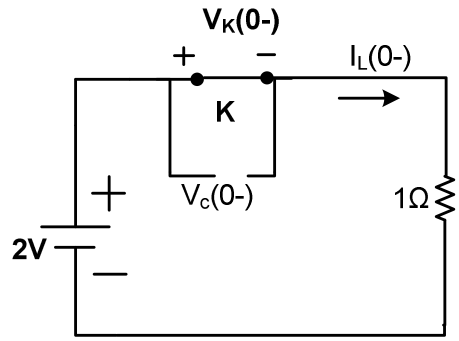


Figure: 89 Network at t = 0-

C gets uncharged as it is short circuited through switch K so
$V_K(0-) = V_c(0-) = 0$
$I_L(0-) = 2/1 = 2A$
Step II: Network at t = 0+
As switch K is opened at t= 0 so it will remain opened at t = 0+
As the Capacitor opposes sudden change in its voltage
Then $V_c(0-) = V_c(0+) = 0$ so that C will be replaced by a short circuit
As the inductor opposes sudden change in its current
Then $I_L(0-) = I_L(0+) = 2A$ so that inductor will be replaced by current source of 2A.
Network at t = 0+ is as shown below

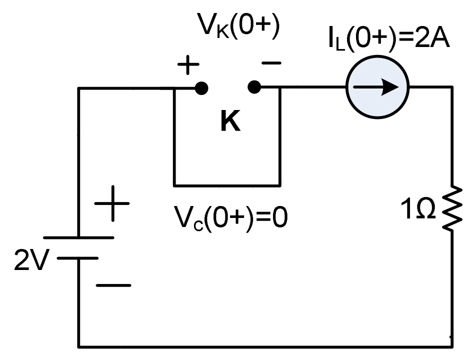


Figure: 88 Network at t = 0+

$V_K(0-) = V_c(0+) = V_c(0-) = 0$
$I_L(0-) = I_L(0+) = 2A$
Step III: Network at t > 0

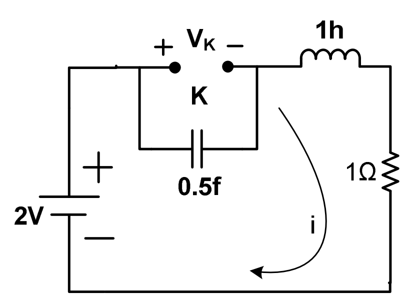


Figure: 89 Network at t > 0

$$V_K = \frac{1}{C}\int idt ----(1)$$

Differentiating equation 1 with respect to time t as

$$\frac{dV_K}{dt} = \frac{i}{C} ----(2)$$

At t = 0+ equation 2 becomes

$$\frac{dV_K}{dt}(0+) = \frac{i(0+)}{C} = \frac{i_L(0+)}{C} = \frac{2}{0.5} = 4V/sec$$

Q6. In the given network, the switch K is opened at t = 0. At t = 0+, solve for the values of v, dv/dt and d^2v/dt^2. if I = 10 A, R = 1000 Ω and C = 1 μF.

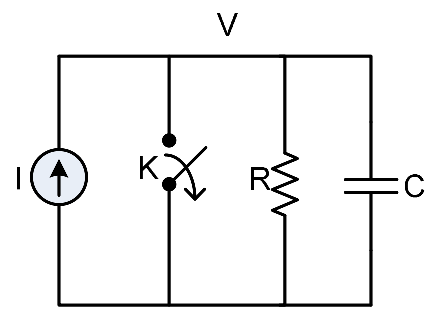


Figure: 90 Q6

Answer:

Step I: Network at t = 0-

As switch K is opened at t= 0 so it will remain opened at t = 0+

C is to be kept open circuit at t = 0- so that voltage across it can be found out

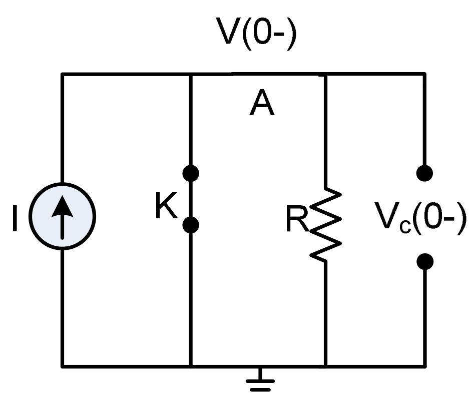


Figure: 91 Network at t = 0-

As the switch is shorted to ground, node A is directly connected to ground so that voltage at A is 0. i.e. V(0-) = 0.

Again current of the current source flows through short circuited switch so that no current flows through R and zero voltage drop across it. As all the components i.e. current source, switch, R and C are connected in parallel in the given network voltage across all of them must be same. So that C is uncharged i.e. $V_c(0-) = 0$.

$V(0-) = V_c(0-) = 0$

Step II: Network at t = 0+
As switch K is opened at t= 0 so it will remain opened at t = 0+
As the Capacitor opposes sudden change in its voltage
Then $V_c(0-) = V_c(0+) = 0$ so that C will be replaced by a short circuit

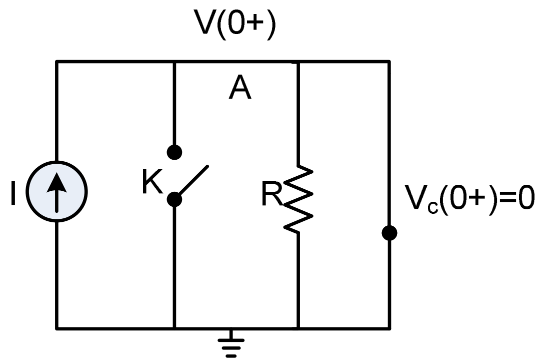


Figure: 92 Network at t = 0+

Now current of the current source flows through short circuited C so that no current flows through R. Voltage across R and C is 0
$V(0+) = V_c(0+) = 0$
Step III: Network at t > 0

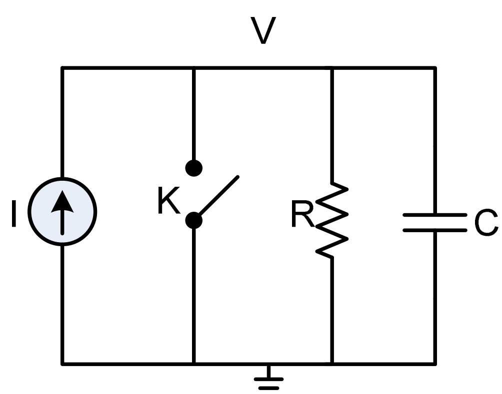


Figure: 93 Network at t > 0

KCL at node A

$$\frac{v}{R} + C\frac{dv}{dt} = I - - - - (1)$$

At t = 0+ equation (1) becomes

$$\frac{v(0+)}{R} + C\frac{dv}{dt}(0+) = I$$

$$\frac{dv}{dt}(0+) = \frac{I - \frac{v(0+)}{R}}{C} = \frac{I}{C} = \frac{10}{1 \times 10^{-6}} = \frac{10^7 V}{sec}$$

Differentiating equation (1)

$$\frac{1}{R}\frac{dv}{dt} + C\frac{d^2v}{dt^2} = \frac{dI}{dt} = 0 - - - - (2)$$

At t = 0+ equation (2) becomes

$$\frac{1}{R}\frac{dv}{dt}(0+) + C\frac{d^2v}{dt^2}(0+) = 0$$
$$\frac{d^2v}{dt^2}(0+) = -\frac{1}{RC}\frac{dv}{dt}(0+) = -\frac{10^7}{1000 \times 1 \times 10^{-6}} = -10^{10}V/sec^2$$

2 CIRCUIT ANALYSIS WITH LAPLACE TRANSFORMS

Q7. For the LC network shown in the Fig., find the transform impedance, Z(s), in the form of a quotient of polynomials, p(s)/q(s). Factorize p(s) and q(s).

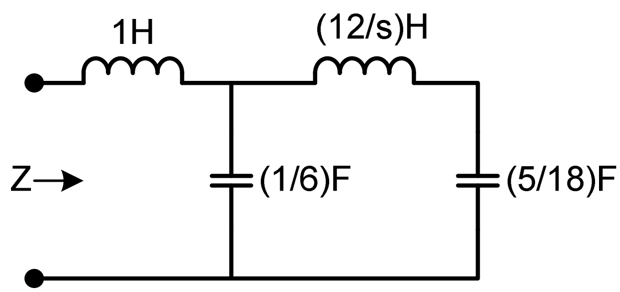


Figure: 94 Q7

Answer:
Transformed network on impedance basis is as shown below

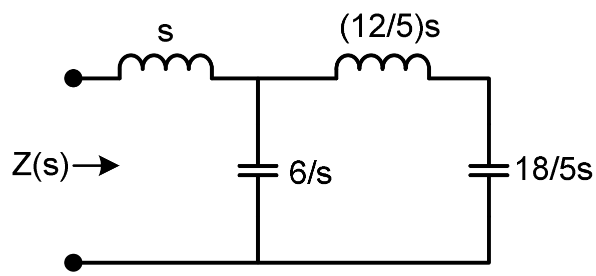


Figure: 95 Transformed network on impedance basis

From the above figure

$$\left(\frac{12s}{5} + \frac{18}{5s}\right) = \frac{(12s \times 5s) + (5 \times 18)}{(5)(5s)} = \frac{60s^2 + 90}{25s}$$

$$\left(\frac{12s}{5} + \frac{18}{5s}\right) || \frac{6}{s} = \frac{\frac{60s^2 + 90}{25s} \times \frac{6}{s}}{\frac{60s^2 + 90}{25s} + \frac{6}{s}} = \frac{3(2s^2 + 3)}{s(s^2 + 4)}$$

$$Z(s) = \left(\frac{12s}{5} + \frac{18}{5s}\right) || \frac{6}{s} + s$$

$$Z(s) = \frac{3(2s^2+3)}{s(s^2+4)} + s$$

$$Z(s) = \frac{s^2 + 10s^2 + 9}{s(s^2+4)}$$

$$Z(s) = \frac{(s^2+1)(s^2+9)}{s(s+2j)(s-2j)}$$

$$Z(s) = \frac{(s+j)(s-j)(s+3j)(s-3j)}{s(s+2j)(s-2j)}$$

Q8. For the RC network shown in the Fig., find the transform impedance, Z(s), in the form of a quotient of polynomials, p(s)/q(s). Factorize p(s) and q(s).

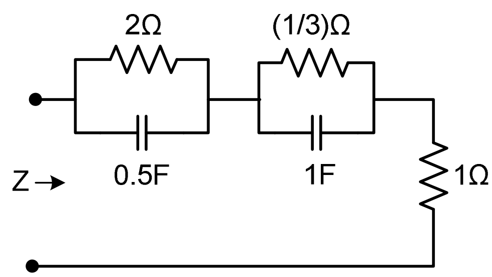


Figure: 96 Q8

Answer:

Transformed network on impedance basis is as shown below

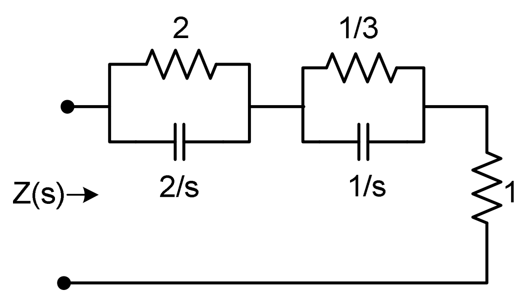


Figure: 97 Transformed network on impedance basis

$$Z(s) = (2||\frac{2}{s}) + (\frac{1}{3}||\frac{1}{s}) + 1$$

$$Z(s) = \frac{2 \times \frac{2}{s}}{2 + \frac{2}{s}} + \frac{\frac{1}{3} \times \frac{1}{s}}{\frac{1}{3} + \frac{1}{s}} + 1$$

$$Z(s) = \frac{2}{s+1} + \frac{1}{s+3} + 1$$
$$Z(s) = \frac{s^2 + 7s + 10}{(s+1)(s+3)}$$
$$Z(s) = \frac{(s+2)(s+5)}{(s+1)(s+3)}$$

Q9. In the network shown in the Fig., C is charged to V_o, and the switch K is closed at t = 0. Solve for the current i(t) using the Laplace transformation method.

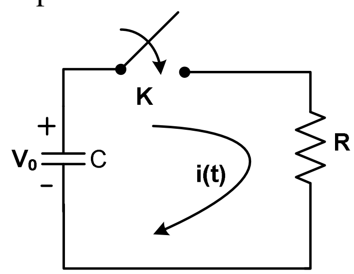


Figure: 98 Q9

Answer:

Step I: Network at t = 0-
As the C is charged to V so
$V_c(0\text{-}) = V_0$
Step II: Transformed network on impedance basis is as shown below

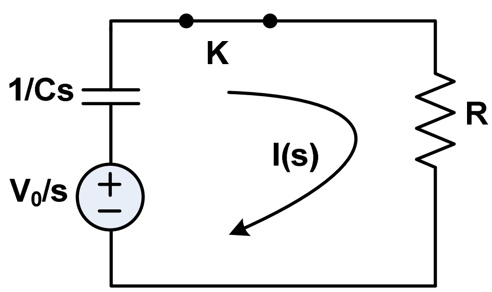


Figure: 99 Transformed network on impedance basis

KVL to above mesh with current I(s)
$$RI(s) + \frac{1}{Cs} I(s) = \frac{V_0}{s}$$
$$I(s) = \frac{V_0}{R} \times \frac{1}{s + \frac{1}{RC}}$$
Take inverse Laplace transform to convert it into time domain
$$i(t) = \frac{V_0}{R} e^{-\frac{t}{RC}}$$

Q10. In the network shown, C is initially charged to V_o. The switch K is closed at t = 0. Solve for the current i(t), using Laplace transformation method.

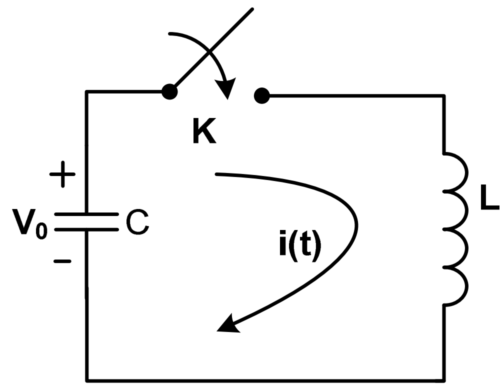


Figure: 100 Q10

Answer:

Step I: Network at t = 0-

As switch K is closed at t= 0 so it will remain opened at t = 0-

To find the current flowing through L at t = 0-, it should be shorted

As the Capacitor opposes sudden change in its voltage and initially it is charged to voltage V_0 it will be replaced by a voltage source at t = 0-.

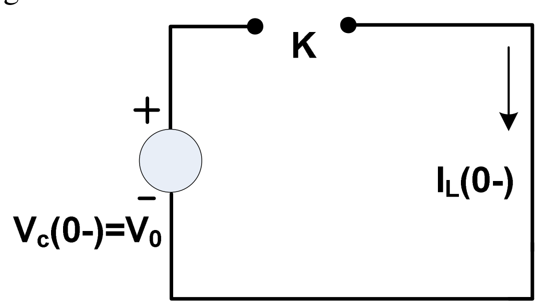


Figure: 101 Network at t = 0-

From the above network

$i_L(0\text{-}) = 0$

Step II: Transformed network on impedance basis is as shown below

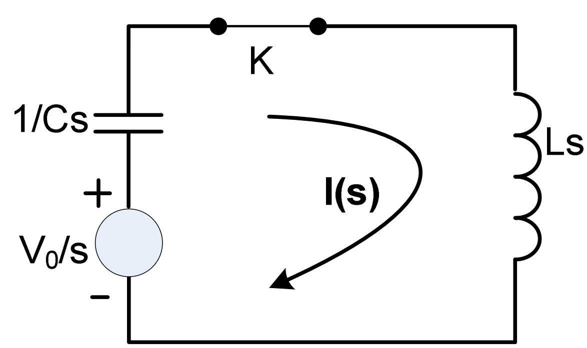


Figure: 102 Transformed network on impedance basis

KVL to the above mesh with current I(s)

$$\left(Ls + \frac{1}{Cs}\right) I(s) = \frac{V_0}{s}$$

$$I(s) = \frac{V_0}{L} \times \frac{1}{s^2 + 1}$$

Take inverse Laplace transform to convert it into time domain

$$i(t) = \frac{V_0}{L} sint$$

References

1. Network Analysis, 3rd Edition M. E. Van Valkenburg PHI Learning Private Limited.
2. Sudhakar, A., Shyammohan, S. P.; "Circuits and Network"; Tata McGraw-Hill New Delhi, 1994.
3. A William Hayt, "Engineering Circuit Analysis" 8th Edition, McGraw-Hill Education.
4. C. K. Alexander and M. N. O. Sadiku, "Fundamentals of Electric Circuits" 5th Edition, McGraw-Hill Education.

UNIT No 5

UNIT-5 : Transforms of other Signal Waveforms, Network Functions, Poles and Zeros of network functions

Unit step, ramp and impulse functions with and without time delay, their Laplace transform, waveform synthesis and its application to electrical networks. Terminal pairs or ports, network functions for one port and two port networks, definition and physical interpretation of poles and zeros, pole-zero plot for network functions, restrictions on pole and zero locations for driving point and transfer functions, time domain behavior from the pole – zero plot, network synthesis using pole – zero plot.

1 WAVEFORM SYNTHESIS

Q1. Write an equation for the waveform, v(t) shown below and determine the Laplace transform of this function.

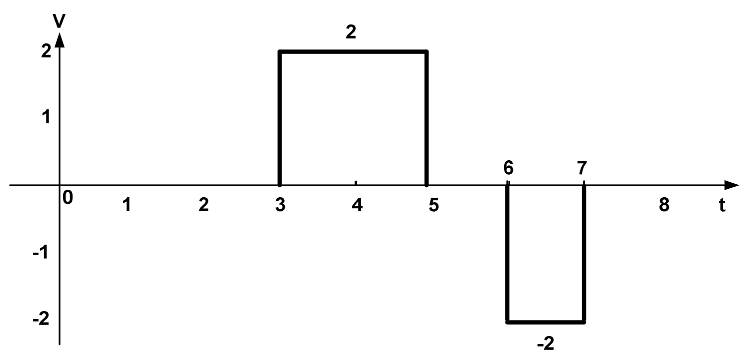


Figure: 103 Q1

Answer:

V(t) = 2u(t - 3) – 2u(t - 5) – 2u(t - 6) + 2u(t - 7)

$$V(s) = \frac{2e^{-3s}}{s} - \frac{2e^{-5s}}{s} - \frac{2e^{-6s}}{s} + \frac{2e^{-7s}}{s}$$

Q2. The waveform shown in Fig. is a terminated truncated ramp. For this v(t), determine the transform V(s).

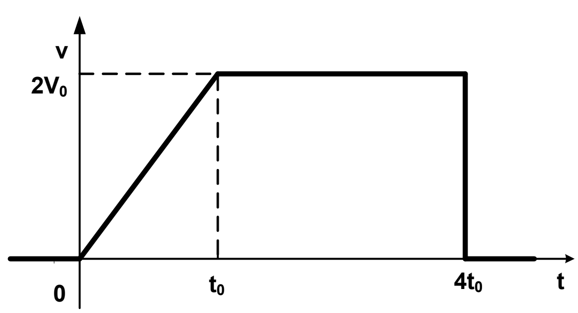


Figure: 104 Q5

Answer:

Slope of the Ramp is =$2V_0$ / t_0

$$v(t) = \frac{2v_0}{t_0}r(t) - \frac{2v_0}{t_0}r(t - t_0) - v_0u(t - 4t_0)$$

$$v(s) = \frac{2v_0}{t_0 s^2} - \frac{2v_0 e^{-t_0 s}}{t_0 s^2} - \frac{2v_0 e^{-4t_0 s}}{s}$$

Q3. The waveform shown in the Fig. is nonrecurring. Write an equation for this waveform, v(t). Also determine the Laplace transform of this function.

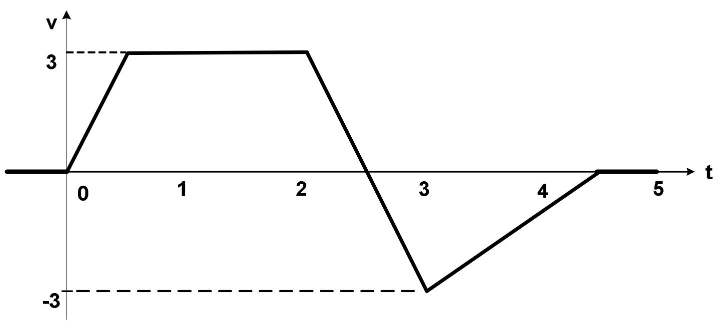


Figure: 105 Q3

The slope is increased by 2 from t = 0.

V(t) = 2r(t)

Stop increased by 2 from t = 1.5.

V(t) = 2r(t) - 2r(t – 1.5)

The slope is decreased by -6 from t = 2.

V(t) = 2r(t) - 2r(t – 1.5) - 6r(t – 2)

Stop increased from t = 3.

V(t) = 2r(t) - 2r(t – 1.5) - 6r(t – 2) + 6r(t – 3)

Slope increased by 2 from t = 3.

V(t) = 2r(t) - 2r(t – 1.5) - 6r(t – 2) + 6r(t – 3) + 2r(t – 3)

Slope increased by 2 from t = 4.5.

V(t) = 2r(t) - 2r(t – 1.5) - 6r(t – 2) + 6r(t – 3) + 2r(t – 3) - 2r(t – 4.5)

V(t) = 2r(t) - 2r(t – 1.5) - 6r(t – 2) + 8r(t – 3) - 2r(t – 4.5)

$$v(s) = \frac{2}{s^2} - \frac{2e^{-1.5s}}{s^2} - \frac{6e^{-2s}}{s^2} + \frac{8e^{-3s}}{s^2} - \frac{2e^{-4.5s}}{s^2}$$

Q4. The waveform shown in the Fig. is nonrecurring. Write an equation for this waveform, v (t). Also determine the Laplace transform of this function.

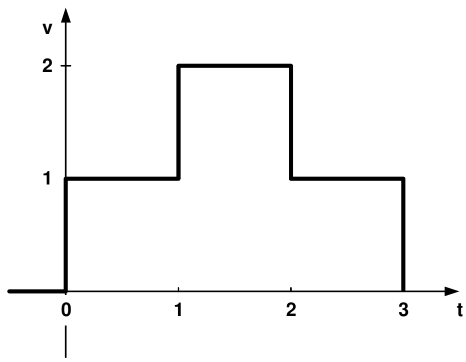


Figure: 106 Q4

Answer:

V(t) = u(t) – u(t - 1) + 2u(t - 1) – 2u(t - 2) + u(t - 2) – u(t - 3)

V(t) = u(t) u(t - 1) + u(t - 2) – u(t - 3)

$$v(s) = \frac{1}{s} + \frac{e^{-s}}{s} - \frac{e^{-2s}}{s} - \frac{e^{-3s}}{s}$$

Q5. The waveform shown in the Fig. consists of a single triangular pulse. For this v(t), determine the corresponding transform V(s).

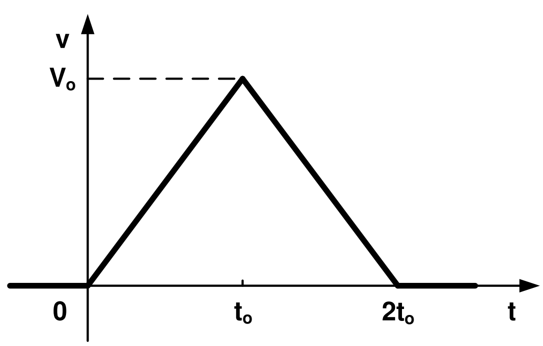


Figure: 107 Q5

Answer:

The slope is increased by V_0/t_0 from t = 0.

$V(t) = (V_0/t_0)r(t)$

Stop increased from $t = t_0$.

$V(t) = (V_0/t_0)r(t) - (V_0/t_0)r(t – t_0)$

The slope is decreased by $(-V_0/t_0)$ from $t = t_0$.

$V(t) = (V_0/t_0)r(t) - (V_0/t_0)r(t - t_0) - (V_0/t_0)r(t - t_0)$

Stop decreased from $t = 2t_0$.

$V(t) = (V_0/t_0)r(t) - (V_0/t_0)r(t - t_0) - (V_0/t_0)r(t - t_0) + (V_0/t_0)r(t - 2t_0)$

$V(t) = (V_0/t_0)r(t) - 2(V_0/t_0)r(t - t_0) + (V_0/t_0)r(t - 2t_0)$

$$V(s) = \frac{v_0}{t_0 s^2} - \frac{2v_0 e^{-t_0 s}}{t_0 s^2} + \frac{v_0 e^{-2t_0 s}}{t_0 s^2}$$

2 NETWORK FUNCTIONS

Q6. For the network shown in the accompanying Fig., Find $Z_{12}(s) = \dfrac{V_2(s)}{I_1(s)}$.

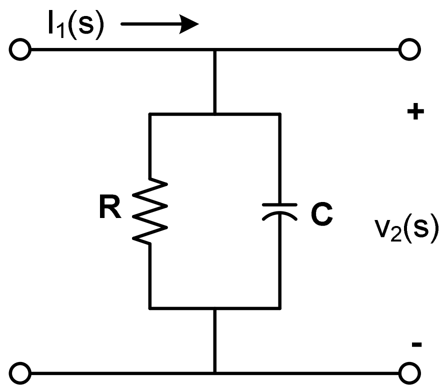


Figure: 108 Q6

Answer:

Transformed network on impedance basis is as shown below

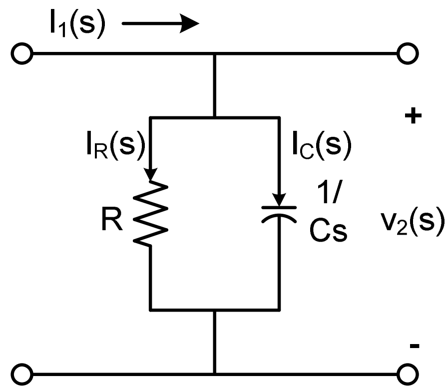


Figure: 109 Transformed network on impedance basis

$$Z_{12}(s) = \frac{V_2(s)}{I_1(s)}$$

$$V_2(s) = R \times I_R(s) = \frac{I_C(s)}{Cs}$$

Using current division rule

$$I_R(s) = I_1(s) \times \frac{\frac{1}{Cs}}{R + \frac{1}{Cs}} = \frac{I_1(s)}{RCs + 1}$$

$$V_2(s) = R \times I_R(s) = R\frac{I_1(s)}{RCs + 1}$$

$$Z_{12}(s) = \frac{V_2(s)}{I_1(s)}$$

$$= \frac{R}{RCs + 1}$$

Q7. The given network contains resistors and controlled sources. For this network, compute

$$G_{12} = \frac{V_2}{V_1}$$

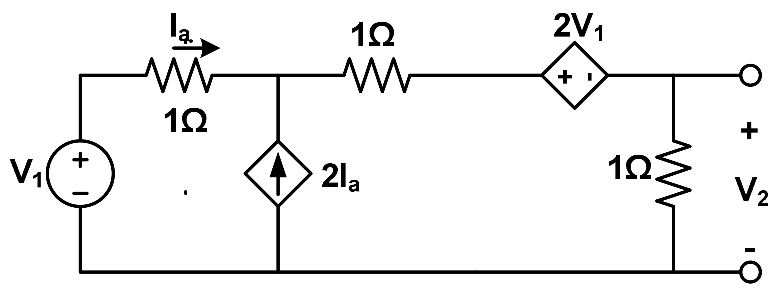


Figure: 110 Q7

Answer:

KVL to supermesh of the above network

$1(I_a) + 1(3I_a) + 2V_1 + 1(3I_a) = V_1$

$I_a + 3I_a + 2V_1 + 3I_a = V_1$

$2V_1 + 7I_a = V_1$

$7I_a = V_1 - 2V_1$

$7I_a = -V_1$ or

$V_1 = -7I_a$

By Ohm's Law

$V_2 = 1(3I_a) = 3I_a$

$$G_{12} = \frac{V_2}{V_1} = \frac{3I_a}{-7I_a} = -\frac{3}{7}$$

Q8. For the network of the accompanying figure and the element values specified, determine

$$\alpha_{12} = \frac{I_2}{I_1}$$

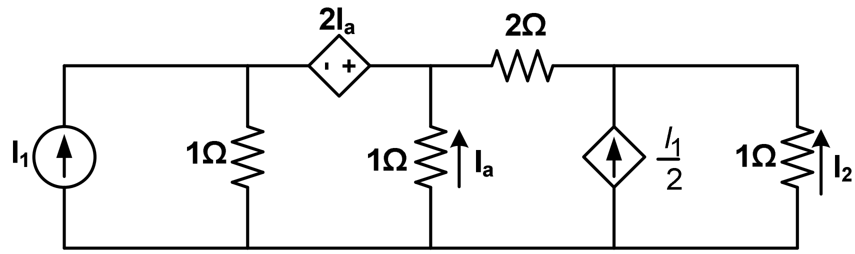


Figure: 111Q8

Answer:

Consider the redrawn network with all currents labeled as follows

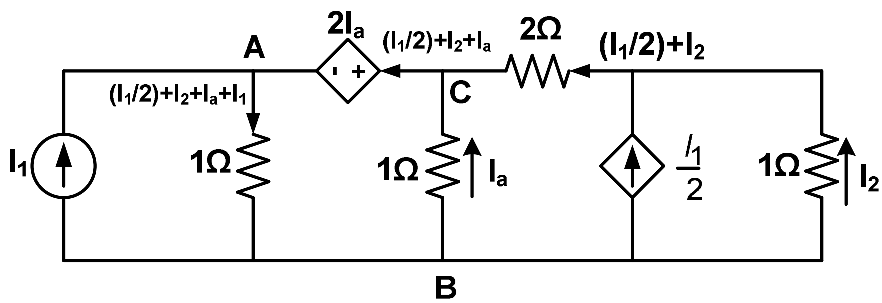


Figure: 112 Network with current directions

KVL to the mesh formed by A-B-C-A

$(1.5I_1 + I_2 + I_a)\,(1) + (I_a)(1) = 2I_a$

$(1.5I_1 + I_2 + I_a)\ + I_a = 2I_a$

$(1.5I_1 + I_2 + I_a)\ = 2I_a - I_a$

$1.5I_1 + I_2\ = 2I_a - I_a - I_a$

$1.5I_1 + I_2\ = 0$

$$\alpha_{12} = \frac{I_2}{I_1} = -1.5$$

Q9. For the network shown in the figure, show that the voltage – ratio transfer function is

$$G_{12} = \frac{(s^2+1)^2}{5s^4+5s^2+1}$$

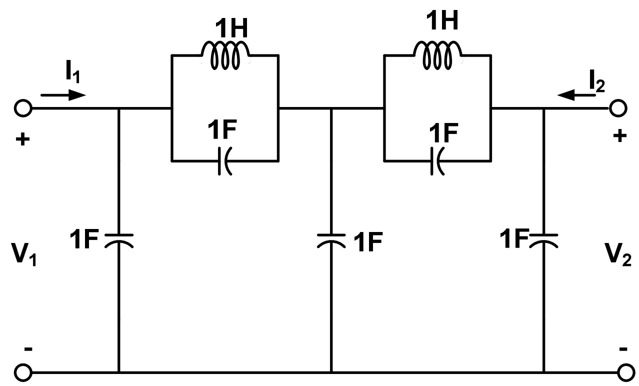


Figure: 113 Q9

Answer:
Transformed network on impedance basis is as shown below

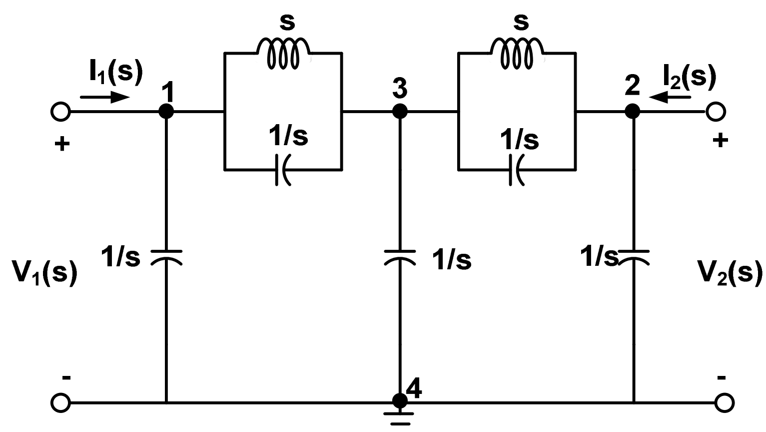


Figure: 114 Transformed network on impedance basis

KCL at node 3

$$\frac{V_3(s)-V_1(s)}{z(s)}+\frac{V_3(s)-V_2(s)}{z(s)}+\frac{V_3(s)}{\frac{1}{s}}=0$$

$$\frac{V_3(s)}{z(s)}-\frac{V_1(s)}{z(s)}+\frac{V_3(s)}{z(s)}-\frac{V_2(s)}{z(s)}+\frac{V_3(s)}{\frac{1}{s}}=0$$

$$\frac{V_1(s)}{z(s)}=\frac{V_3(s)}{z(s)}+\frac{V_3(s)}{z(s)}-\frac{V_2(s)}{z(s)}+\frac{V_3(s)}{\frac{1}{s}}----(1)$$

Where

$$z(s)=s||\frac{1}{s}=\frac{s\times\frac{1}{s}}{s+\frac{1}{s}}=\frac{s}{s^2+1}$$

KCL at node 2

$$\frac{V_2(s)-V_3(s)}{z(s)}+\frac{V_2(s)}{\frac{1}{s}}=0$$

$$\frac{V_2(s)}{z(s)} - \frac{V_3(s)}{z(s)} + \frac{V_2(s)}{\frac{1}{s}} = 0$$

$$\frac{V_3(s)}{z(s)} = \frac{V_2(s)}{z(s)} + \frac{V_2(s)}{\frac{1}{s}} ----(2)$$

Where

$$z(s) = s||\frac{1}{s} = \frac{s \times \frac{1}{s}}{s + \frac{1}{s}} = \frac{s}{s^2 + 1}$$

Substituting $V_3(s)$ from equation (2) in equation (1)

$$G_{12}(s) = \frac{V_2(s)}{V_1(s)} = \frac{\frac{1}{z(s)}}{\left[\frac{2}{z(s)} + s\right][1 + sz(s)] - \frac{1}{z(s)}}$$

Substituting z(s) in above equation will get

$$G_{12} = \frac{(s^2+1)^2}{5s^4 + 5s^2 + 1}$$

Q10. For the given network, show that

$$Y_{12}(s) = \frac{K\,(s+1)}{(s+2)\,(s+4)}$$

and determine the value and sign of K

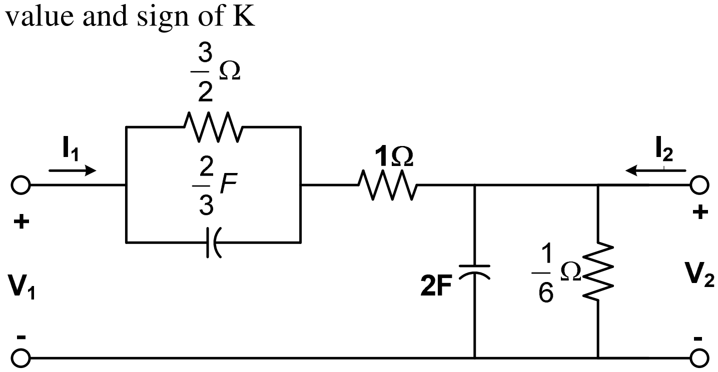


Figure: 115 Q10

Answer:

Transformed network on impedance basis is as shown below

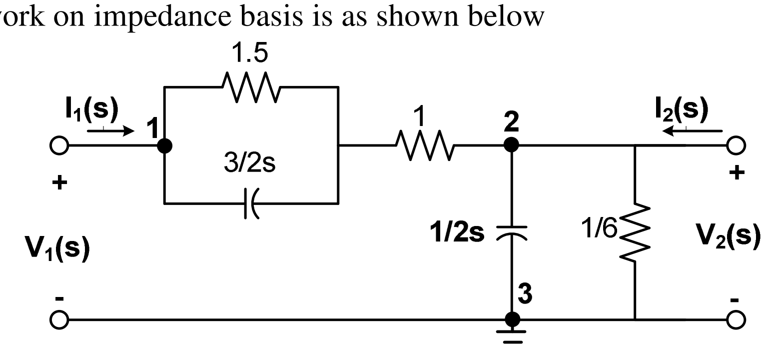


Figure: 116 Transformed network on impedance basis

KCL at node 2

$$\frac{V_2(s) - V_1(s)}{z(s)} + \frac{V_2(s)}{\frac{1}{6}} + \frac{V_2(s)}{\frac{1}{2s}} = 0$$

$$\frac{V_2(s)}{z(s)} - \frac{V_1(s)}{z(s)} + \frac{V_2(s)}{\frac{1}{6}} + \frac{V_2(s)}{\frac{1}{2s}} = 0$$

$$\frac{V_1(s)}{z(s)} = \frac{V_2(s)}{z(s)} + \frac{V_2(s)}{\frac{1}{6}} + \frac{V_2(s)}{\frac{1}{2s}} ----(1)$$

Where

$$z(s) = \frac{3}{2} || \frac{3}{2s} = \frac{\frac{3}{2} \times \frac{3}{2s}}{\frac{3}{2} + \frac{3}{2s}} = \frac{2s+5}{2s+2}$$

Using ohm's law

$$V_2(s) = -\frac{I_2(s)}{6} ----(2)$$

Substituting $V_2(s)$ from equation (2) in equation (1)

$$\frac{V_1(s)}{z(s)} = -\left[\frac{1}{z(s)} + \frac{1}{\frac{1}{6}} + \frac{1}{\frac{1}{2s}}\right] \frac{I_2(s)}{6}$$

$$Y_{12}(s) = \frac{I_2(s)}{V_1(s)} = \frac{\frac{1}{z(s)}}{-\left[\frac{1}{z(s)} + \frac{1}{\frac{1}{6}} + \frac{1}{\frac{1}{2s}}\right] \frac{1}{6}} = \frac{6}{(2s + 6z)z(s) + 1}$$

Substituting z(s) in above equation will get

$$Y_{12}(s) = -\frac{3(s+1)}{(s+2)(s+4)} = \frac{K(s+1)}{(s+2)(s+4)}$$

Therefore K = -3

References

1. Network Analysis, 3rd Edition M. E. Van Valkenburg PHI Learning Private Limited.
2. Sudhakar, A., Shyammohan, S. P.; "Circuits and Network"; Tata McGraw-Hill New Delhi, 1994.
3. A William Hayt, "Engineering Circuit Analysis" 8th Edition, McGraw-Hill Education.
4. C. K. Alexander and M. N. O. Sadiku, "Fundamentals of Electric Circuits" 5th Edition, McGraw-Hill Education.

UNIT No 6

UNIT-6: Two Port Parameters

Standard reference directions for the voltages and currents of a two – port network, defining equations for open circuit impedance, , transmission, inverse transmission, hybrid and inverse hybrid parameters, relationships between parameter sets, conditions for reciprocity and electrical symmetry in terms of two – port parameters, , interconnections of two - port networks.

1 TWO PORT PARAMETERS

Q1. Find the y and z parameters for the network shown in the Fig.

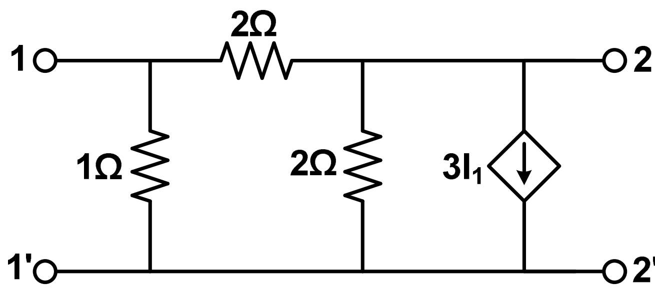


Figure: 117 Q1

Answer:

Given two port network with all current directions and voltage polarities is shown below

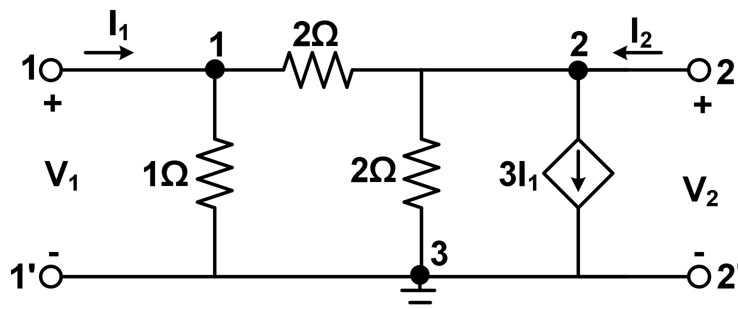


Figure: 118 Network with current directions and voltage polarities

KCL at node 1 is

$$I_1 = \frac{V_1}{1} + \frac{V_1 - V_2}{2}$$

$$I_1 = \frac{V_1}{1} + \frac{V_1}{2} - \frac{V_2}{2}$$

$$I_1 = V_1\left[\frac{1}{1} + \frac{1}{2}\right] - \frac{V_2}{2}$$

$$I_1 = 1.5V_1 - 0.5V_2 ----(1)$$

KCL at node 2 is

$$I_2 = 3I_1 + \frac{V_2}{2} + \frac{V_2 - V_1}{2}$$

$$I_2 = 3I_1 + \frac{V_2}{2} + \frac{V_2}{2} - \frac{V_1}{2}$$
$$I_2 = 3I_1 + \frac{V_2}{2} + \frac{V_2}{2} - \frac{V_1}{2}$$
$$I_2 = 3I_1 + V_2 - 0.5V_1$$

Substituting I_1 from equation (1)

$$I_2 = 3(1.5V_1 - 0.5V_2) + V_2 - 0.5V_1$$
$$I_2 = 4.5V_1 - 1.5V_2 + V_2 - 0.5V_1$$
$$I_2 = 4V_1 - 0.5V_2 ----(2)$$

Equations (1) and (2) in matrix form

$$\begin{bmatrix} 1.5 & -0.5 \\ 4 & -0.5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Y-parameters are defined by

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$
$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 1.5 & -0.5 \\ 4 & -0.5 \end{bmatrix}$$
$$[Z] = [Y]^{-1} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} -0.4 & 0.4 \\ -3.2 & 1.2 \end{bmatrix}$$

Q2. Find the y and z parameters for the network shown in the Fig.

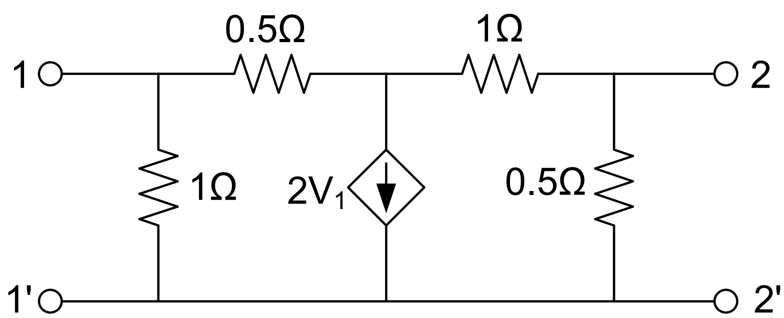
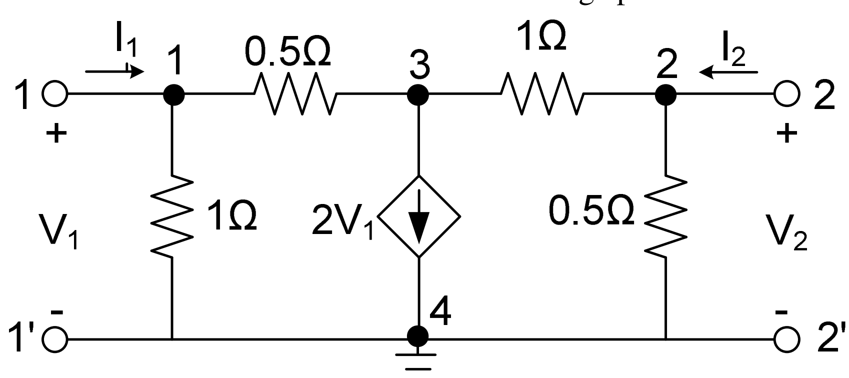


Figure: 119 Q2

Answer:

Given two port network with all current directions and voltage polarities is shown below



KCL at node 1 is

$$\frac{V_1 - V_3}{0.5} + \frac{V_1}{1} = I_1$$
$$\frac{V_1}{0.5} - \frac{V_3}{0.5} + \frac{V_1}{1} = I_1$$
$$\frac{V_1}{0.5} - \frac{V_3}{0.5} + \frac{V_1}{1} = I_1$$

$3V_1$ - $2V_3$ = I_1-------(1)

KCL at node 2 is

$$\frac{V_2 - V_3}{1} + \frac{V_2}{0.5} = I_2$$
$$\frac{V_2}{1} - \frac{V_3}{1} + \frac{V_2}{0.5} = I_2$$

$3V_2$ - V_3 = I_2-------(2)

KCL at node 3 is

$$\frac{V_3 - V_1}{0.5} + \frac{V_3 - V_2}{1} + 2V_1 = 0$$
$$\frac{V_3}{0.5} - \frac{V_1}{0.5} + V_3 - V_2 + 2V_1 = 0$$
$$2V_3 - 2V_1 + V_3 - V_2 + 2V_1 = 0$$
$$3V_3 - V_2 = 0$$
$$3V_3 = V_2$$
$$V_3 = \frac{V_2}{3} - - - - (3)$$

Substituting V_3 from equation (3) in equation (1)

$3V_1$ - $2V_3$ = I_1-------(1)

$3V_1$ - (2/3) V_2 = I_1

Substituting V_3 from equation (3) in equation (2)

$3V_2$ - V_3 = I_2-------(2)

$3V_2$ – (V_2/3) = I_2

$I_2 = \frac{8}{3}V_2$-------(2)

Equations (1) and (2) in matrix form

$$\begin{bmatrix} 3 & -2/3 \\ 0 & 8/3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Y-parameters are defined by

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$
$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 3 & -2/3 \\ 0 & 8/3 \end{bmatrix}$$
$$[Z] = [Y]^{-1} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 0.333 & 0.083 \\ 0 & 0.375 \end{bmatrix}$$

Q3. Derive condition for electrical symmetry in terms of Z - parameters.
Answer:
Z – Parameters are given as
$V_1 = Z_{11}I_1 + Z_{12}I_2$----(1)
$V_2 = Z_{21}I_1 + Z_{22}I_2$----(2)
Consider port 2 as open so that current $I_2 = 0$ equation (1) will become
$V_1 = Z_{11}I_1$
$$\frac{V_1}{I_1} = Z_{11}\, with\, I_2 = 0$$
$$\frac{I_1}{V_1} = \frac{1}{Z_{11}} - - - - (3)$$
Consider port 1 as open so that current $I_1 = 0$ equation (2) will become
$V_2 = Z_{22}I_2$
$$\frac{V_2}{I_2} = Z_{22}\, with\, I_1 = 0$$
$$\frac{I_2}{V_2} = \frac{1}{Z_{22}} - - - - (4)$$
From equations (3) and (4) condition for electrical symmetry in terms of Z – parameters is
$$\frac{1}{Z_{11}} = \frac{1}{Z_{22}}$$
Or
$Z_{11} = Z_{22}$

Q4. Derive condition for electrical symmetry in terms of Y - parameters.
Answer:
Y– Parameters are given as
$I_1 = Y_{11}V_1 + Y_{12}V_2$----(1)
$I_2 = Y_{21}V_1 + Y_{22}V_2$----(2)
Consider port 2 as open so that current $I_2 = 0$ equation (2) will become
$0 = Y_{21}V_1 + Y_{22}V_2$
$Y_{21}V_1 = - Y_{22}V_2$
$$-\frac{Y_{21}}{Y_{22}}V_1 = V_2$$
Substituting V_2 from above equation in equation (1)
$I_1 = Y_{11}V_1 + Y_{12}V_2$
$I_1 = Y_{11}V_1 + Y_{12}(-\frac{Y_{21}}{Y_{22}}V_1)$
$$\frac{I_1}{V_1} = \frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}}\; with\; I_2 = 0 - - - - (3)$$
Consider port 1 as open so that current $I_1 = 0$ equation (2) will become
$0 = Y_{11}V_1 + Y_{12}V_2$----(1)
$Y_{11}V_1 = - Y_{12}V_2$
$$-\frac{Y_{12}}{Y_{11}}V_2 = V_1$$
Substituting V_1 from above equation in equation (2)
$I_2 = Y_{21}V_1 + Y_{22}V_2$

$I_2 = Y_{21}[-\frac{Y_{12}}{Y_{11}}V_2] + Y_{22}V_2$

$$\frac{I_2}{V_2} = \frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{22}}\ with\ I_1 = 0 - - - - (4)$$

From equations (3) and (4) condition for electrical symmetry in terms of Y – parameters is

$$\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}} = \frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{22}}$$

Or

$Y_{11} = Y_{22}$

Q5. Derive condition for electrical symmetry in terms of T - parameters.

Answer:

T – Parameters are given as

$V_1 = AV_2 - BI_2$----(1)

$I_1 = CV_2 - DI_2$----(2)

Consider port 2 as open so that current $I_2 = 0$ equation (1) and (2) will become

$V_1 = AV_2$

$I_1 = CV_2$

$$\frac{I_1}{V_1} = \frac{C}{A}\ with\ I_2 = 0 - - - - (3)$$

Consider port 1 as open so that current $I_1 = 0$ equation (1) and (2) will become

$V_1 = BI_2$

$I_1 = DI_2$

$$\frac{I_2}{V_2} = \frac{C}{D}\ with\ I_1 = 0 - - - - (4)$$

From equations (3) and (4) condition for electrical symmetry in terms of T – parameters is

$$\frac{C}{A} = \frac{C}{D}$$

Or

$A = D$

Q6. Derive condition for electrical symmetry in terms of T' - parameters.

Answer:

T' – Parameters are given as

$V_2 = A'V_1 – B'I_1$----(1)

$I_2 = C'V_1 – D'I_1$----(2)

Consider port 2 as open so that current $I_2 = 0$ equation (2) will become

$C'V_1 = D'I_1$

$$\frac{I_1}{V_1} = \frac{C'}{D'}\ with\ I_2 = 0 - - - - (3)$$

Consider port 1 as open so that current $I_1 = 0$ equation (1) and (2) will become

$V_2 = A'V_1$

$I_2 = C'V_1$

$$\frac{I_2}{V_2} = \frac{C'}{A'}\ with\ I_1 = 0 - - - - (4)$$

From equations (3) and (4) condition for electrical symmetry in terms of T' – parameters is

$$\frac{C'}{A'} = \frac{C'}{D'}$$

Or

A' = D'

Q7. Derive condition for reciprocity in terms of Z - parameters.

Answer:

Z – Parameters are given as

$V_1 = Z_{11}I_1 + Z_{12}I_2$----(1)

$V_2 = Z_{21}I_1 + Z_{22}I_2$----(2)

Consider port 2 short so that voltage $V_2 = 0$ equation (2) will become

$0 = Z_{21}I_1 + Z_{22}I_2$----(2)

$Z_{21}I_1 = -Z_{22}I_2$

$I_1 = (-Z_{22}/ Z_{21})I_2$

Substituting I_1 in equation (1)

$V_1 = Z_{11}((-Z_{22}/ Z_{21})I_2) + Z_{12}I_2$

$$\frac{I_2}{V_1} = \frac{Z_{21}}{Z_{12}Z_{21} - Z_{11}Z_{22}} \; with \; V_2 = 0 - - - - (3)$$

Consider port 1 short so that voltage $V_1 = 0$ equation (2) will become

$0 = Z_{11}I_1 + Z_{12}I_2$----(1)

$Z_{11}I_1 =- Z_{12}I_2$

$I_2 = (-Z_{11}/ Z_{12})I_1$

Substituting I_2 in equation (2)

$V_2 = Z_{21}I_1 + Z_{22}((-Z_{11}/ Z_{12})I_1)$----(2)

$$\frac{I_1}{V_2} = \frac{Z_{12}}{Z_{12}Z_{21} - Z_{11}Z_{22}} \; with \; V_1 = 0 - - - - (4)$$

From equations (3) and (4) condition for reciprocity in terms of Z – parameters is

$$\frac{Z_{21}}{Z_{12}Z_{21} - Z_{11}Z_{22}} = \frac{Z_{12}}{Z_{12}Z_{21} - Z_{11}Z_{22}}$$

Or

$Z_{12} = Z_{21}$

Q8. Derive condition for reciprocity in terms of Y - parameters.

Answer:

Y– Parameters are given as

$I_1 = Y_{11}V_1 + Y_{12}V_2$----(1)

$I_2 = Y_{21}V_1 + Y_{22}V_2$----(2)

Consider port 2 short so that voltage $V_2 = 0$ equation (2) will become

$I_2 = Y_{21}V_1$

$$\frac{I_2}{V_1} = Y_{21} \; with \; V_2 = 0 - - - - (3)$$

Consider port 1 short so that voltage $V_1 = 0$ equation (1) will become

$I_1 = Y_{12}V_2$

$$\frac{I_1}{V_2} = Y_{12} \; with \; V_1 = 0 - - - - (3)$$

From equations (3) and (4) condition for reciprocity in terms of Y – parameters is
$Y_{12} = Y_{21}$

Q9. Derive condition for reciprocity in terms of T - parameters.
Answer:
T – Parameters are given as
$V_1 = AV_2 - BI_2$----(1)
$I_1 = CV_2 - DI_2$----(2)
Consider port 2 short so that voltage $V_2 = 0$ equation (1) will become
$V_1 = BI_2$
$$\frac{I_2}{V_1} = -\frac{1}{B} \ with\ V_2 = 0 - - - - (3)$$
Consider port 1 short so that voltage $V_1 = 0$ equation (1) and (2) will become
$0 = AV_2 - BI_2$
$AV_2 = BI_2$
$I_2 = (A/B)V_2$
Substituting I_2 in equation (2)
$I_1 = CV_2 – D((A/B)V_2)$----(2)
$$\frac{I_1}{V_2} = \frac{BC - AD}{B} \ with\ V_1 = 0 - - - - (4)$$
From equations (3) and (4) condition for reciprocity in terms of T – parameters is
$$\frac{BC - AD}{B} = -\frac{1}{B}$$
Or AD-BC = 1
Or $\Delta_T = 1$

Q10.Derive condition for reciprocity in terms of T - parameters.
Answer:
T' – Parameters are given as
$V_2 = A'V_1 – B'I_1$----(1)
$I_2 = C'V_1 – D'I_1$------(2)
Consider port 2 short so that voltage $V_2 = 0$ equation (1) and (2) will become
$0 = A'V_1 – B'I_1$----(1)
$A'V_1 = B'I_1$
$I_1 = (A'/B')V_1$
Substituting I_2 in equation (2)
$I_2 = C'V_1 – D'((A'/B')V_1)$----(2)
$$\frac{I_2}{V_1} = \frac{B'C' - A'D'}{B'} \ with\ V_2 = 0 - - - - (3)$$
Consider port 1 short so that voltage $V_1 = 0$ equation (1) will become
$V_2 = – B'I_1$----(1)
$$\frac{I_1}{V_2} = -\frac{1}{B'} \ with\ V_1 = 0 - - - - (3)$$
From equations (3) and (4) condition for reciprocity in terms of T' – parameters is
$$\frac{B'C' - A'D'}{B'} = -\frac{1}{B'}$$

Or $A'D' - B'C' = 1$
Or $\Delta_{T'} = 1$

References

1. Network Analysis, 3rd Edition M. E. Van Valkenburg PHI Learning Private Limited.
2. Sudhakar, A., Shyammohan, S. P.; "Circuits and Network"; Tata McGraw-Hill New Delhi, 1994.
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4. C. K. Alexander and M. N. O. Sadiku, "Fundamentals of Electric Circuits" 5th Edition, McGraw-Hill Education.