Engineering Notebook

VOLUME 1

EE1409 : Digital Communication

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ACKNOWLEDGMENTS

The writing of this notebook was a big task as it consists of lot of mathematical terms and derivations. Fortunately, we had the fine support of our family, teaching staff members, and our Head of Department Dr. P. T. Karule sir.

UNIT No. I

WAVEFORM CODING TECHNIQUES:

Q1. *Explain PCM System with the help of block diagram.*

Answer:

Pulse Code Modulation :

In Pulse Code Modulation, the message signal is represented by a sequence of coded pulses which is the signal in discrete form in both time and amplitude. The Basic elements of a PCM system are shown in figure 1.1.

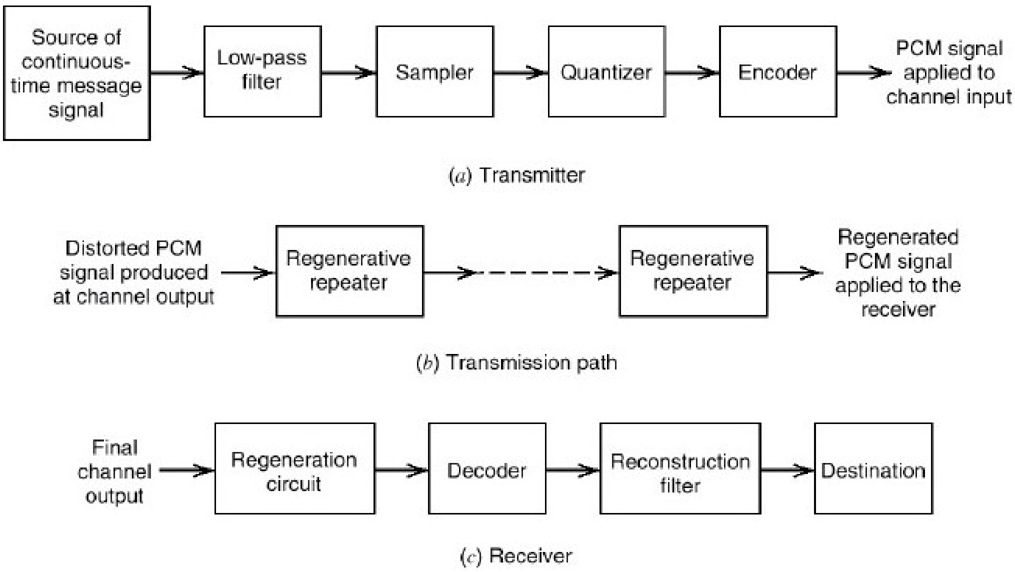


Figure 1.1 : Basic elements of a PCM system a)Transmitter b)Transmission Path c) Receiver

a) PCM Transmitter

The Pulse Code Modulation (PCM) transmitter Section consists of Sampling, Quantizing and Encoding, which performed the analog-to-digital conversion.

i) Low Pass Filter (LPF)

The function of Low Pass filter is to eliminate the high frequency components present in the input analog message signal to avoid aliasing of the message signal.

ii)Sampler

The sampler converts continuous time signal into discrete time signal. In this process analog signal is converted into corresponding sequence samples that usually spaced uniformly in time. The sampling rate must be greater than twice the highest frequency component of the message signal as per the sampling theorem.

iii) Quantizer

The Quantizing is a process of transforming sampled amplitude value of a message signal into discrete amplitude value(levels). The Quantizer approximates each of input sampled value to nearest prefixed level. The Quantizer reduces the redundant bits and compresses the value.

iv) Encoder
Encoding is the process of converting data from one form to another. The Encoder converts each quantized level by a binary code. The digitization of analog signal is done by the encoder. Encoding minimizes the bandwidth used in PCM.

b) Regenerative Repeater
A regenerative repeater amplifies and reconstructs distorted digital signal and develops a nearly perfect replica of the original at its output. Regenerative Repeater section increases strength of the signal. The output of the channel also has one regenerative repeater circuit, to compensate the signal loss and reconstruct the signal .Regenerative repeaters are used during many pulse retransmissions in order to improve the quality of data transmission.

c) PCM Receiver

The PCM receiver Section consists of regeneration of impaired signals, decoding, and reconstruction of the quantized pulse train.

i) Regenerative Repeater

The first part of the receiver is Regenerative Repeater which reshape and clean-up the received pulse one last time.
ii) Decoder

The decoder circuit decodes the pulse coded waveform to reproduce the original signal. This circuit acts as the demodulator. The clean pulses are regrouped into codeword and decoded into a quantized PAM signal.

iii) Reconstruction Filter
Final operation of the receiver is to the message signal by passing decoder output through a low pass reconstruction filter whose cut-off frequency is equal to message bandwidth. The recovered signal is assumed to be error free.

Q2. *Discuss in detail about the operation of Differential PCM Transmitter and Receiver with neat block diagrams.*

Answer:
In the pulse code modulator each sample of waveform is encoded independently of other sample. The samples of the signals are highly correlated with each other. When these samples are encoded by PCM, the resulting encoded signal contains redundant information.

Principle of Differential Pulse Code Modulation (DPCM):

If the redundancy is reduced, then the overall bit rate will decrease and the number of bits required to transmit one sample will also reduce. This type of digital pulse modulation technique is called differential pulse code modulation. The DPCM works on the principle of prediction. The value of the present sample is predicted from the previous samples. The prediction may not be exact, but it is very close to the actual sample value.

Differential Pulse Code Modulation Transmitter:

The Transmitter Section of DPCM consists of Quantizer ,Encoder and Prediction Filter with two summer circuits as shown in Figure 1.2 of DPCM transmitter.

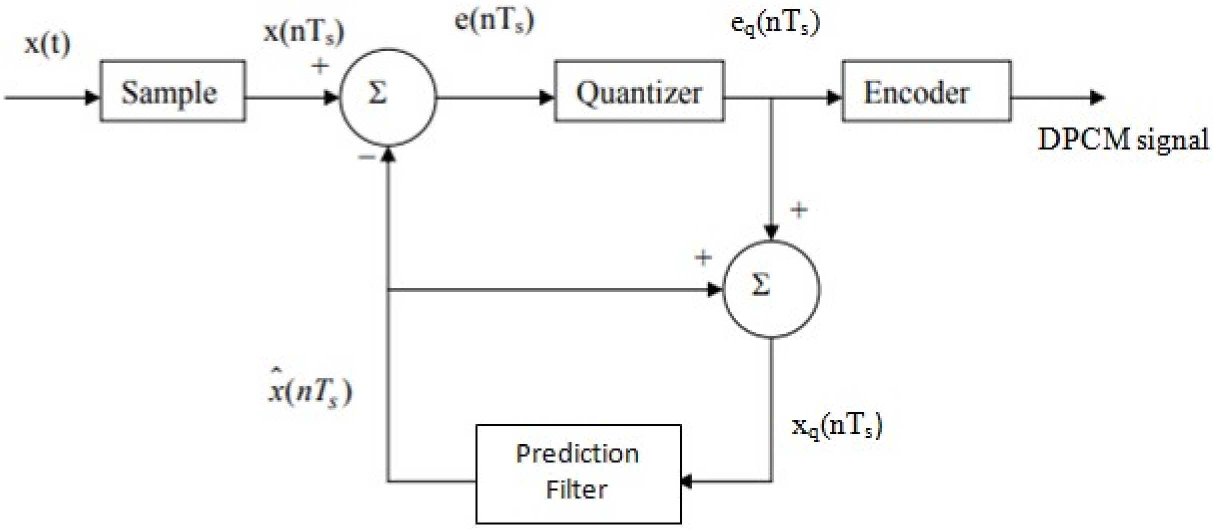


Figure 1.2: Differential Pulse Code Modulation Transmitter

The transmitter consists of a comparator, quantizer, prediction filter, and an encoder. The sampled signal is denoted by $x(nTs)$ and the predicted signal is indicated by $\hat{x}(nTs)$. The comparator finds out the difference between the actual sample value $x(nTs)$ and the predicted value $\hat{x}(nTs)$. This is called signal error and it is denoted as $e(nTs)$.

$$e(nTs) = x(nTs) - \hat{x}(nTs) \qquad (1)$$

The predicted value $\hat{x}(nTs)$ is produced by using a prediction filter. The quantizer output signal $eq(nTs)$ and the previous prediction is added and given as input to the prediction filter, this signal is denoted by $xq(nTs)$. This makes the prediction closer to the actually sampled signal. The quantized error signal $eq(nTs)$ is very small and can be encoded by using a small number of bits. Thus the number of bits per sample is reduced in DPCM. The quantizer output would be written as:

$$eq(nTs) = e(nTs) + q(nTs) \qquad (2)$$

Where $q(nTs)$ is quantization error. From the above block diagram the prediction filter input $xq(nTs)$ is obtained by sum of $\hat{x}(nTs)$ and the quantizer output $eq(nTs)$.

$$xq(nTs) = \hat{x}(nTs) + eq(nTs) \qquad (3)$$

Now substitute the value of $eq(nTs)$ from the equation (2) in equation (3) we get,

$$xq(nTs) = \hat{x}(nTs) + e(nTs) + q(nTs) \qquad (4)$$

Equation (1) can be written as,

$$e(nTs) + \hat{x}(nTs) = x(nTs) \qquad (5)$$

from the above equations 4 and 5 we get,

$$xq(nTs) = x(nTs) + q(nTs) \qquad (6)$$

Therefore, the quantized version of signal $xq(nTs)$ is the sum of original sample value and quantized error $q(nTs)$. The quantized error can be positive or negative. So the output of the prediction filter does not depend on its characteristics.

Differential Pulse Code Modulation Receiver:

In order to reconstruct the received digital signal, the DPCM receiver (shown in the below figure) consists of a decoder and prediction filter. In the absenteeism of noise, the encoded receiver input will be the same as the encoded transmitter output. DPCM is same as the PCM technique used for remodelling analog signal into digital signal. DPCM has moderate signal to noise ratio. DPCM differs from PCM as a result of it quantizes the distinction of the particular sample and expected price. that's the explanation it's referred to as differential PCM.

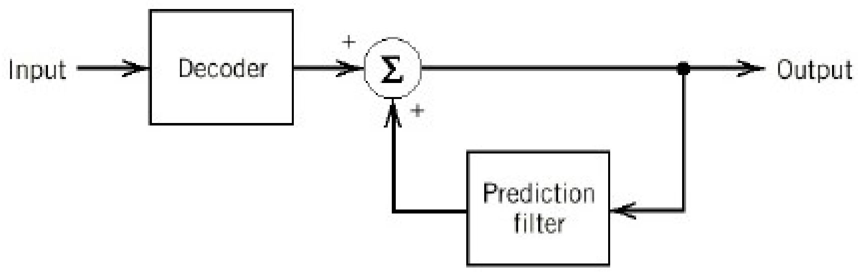


Figure 1.3: Differential Pulse Code Modulation Receiver

Q3.*Explain transmitter & Receiver section for Delta modulation with mathematical analysis. Also state advantages and disadvantages*.

Answer:

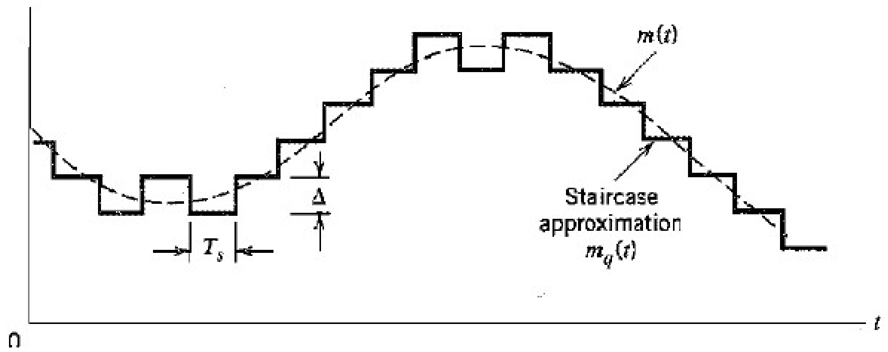
Delta Modulation

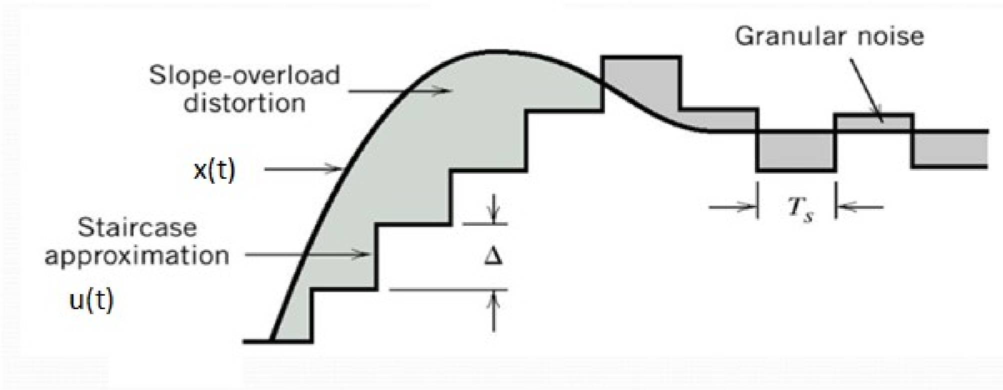
In PCM the signaling rate and transmission channel bandwidth are quite large since it transmits all the bits which are used to code a sample. To overcome this problem, Delta modulation is used.

Working Principle

Delta modulation transmits only one bit per sample. Here, the present sample value is compared with the previous sample value and this result whether the amplitude is increased or decreased is transmitted.

Input signal x(t) is approximated to step signal by the delta modulator.This step size is kept fixed.The difference between the input signal x(t) and staircase approximated signal is confined to two levels, i.e., +Δ and -Δ.Now, if the difference is positive, then approximated signal is increased by one step, i.e., 'Δ'. If the difference is negative, then approximated signal is reduced by 'Δ' .When the step is reduced, '0' is transmitted and if the step is increased, '1' is transmitted.Hence, for each sample, only one binary bit is transmitted. Figure 1.4 shows the analog signal x(t) and its staircase approximated signal by the delta modulator.





Mathematical Analysis

The error between the sampled value of x(t) and last approximated sample is given as:

$$e(nTs) = x(nTs) - \hat{x}(nTs) \qquad (1)$$

Where

$e(nTs)$ -error at present sample.

$x(nTs)$ -Sampled signal at x(t)

$\hat{x}(nTs)$ -last staircase approximation of staircase waveform

If we assume $u(nT_s)$ as the present sample approximation of staircase output, then

$u[(n-1)Ts] = \hat{x}(nTs)$-Last sample approximation of staircase waveform

Let us define a quantity b(nT_s) in such a way that,

$$b(nTs) = \Delta sgn[e(nTs)] \tag{2}$$

This means that depending on the sign of error e(nT_s) , the sign of step size Δ is decided. In other words we can write

$$b(nTs) = \begin{cases} +\Delta & \text{if } x(nTs) \geq \hat{x}(nTS) \\ -\Delta & \text{if } x(nTs) \leq \hat{x}\,(nTs) \end{cases} \tag{3}$$

if $b(nTs) = +\Delta$ then binary 1 is transmitted.

and if $b(nTs) = -\Delta$ then binary 0 is transmitted.

Here T_s = sampling interval.

Delta Modulation Transmitter:

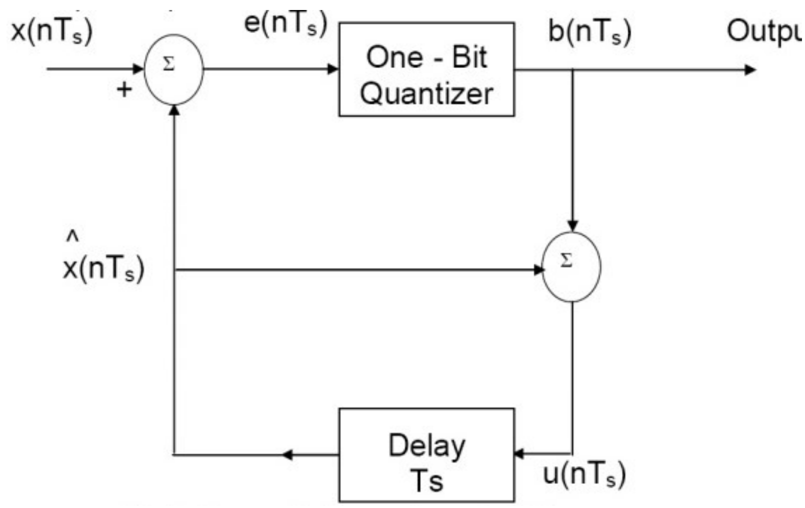


Figure 1.4 : Delta Modulation Transmitter

Figure 1.4 shows the transmitter . It is also known as Delta modulator.It consists of a one-bit quantizer and a delay circuit along with two summer circuits.
The summer in the accumulator adds quantizer output ($\pm\Delta$) with the previous sample approximation . This gives present sample approximation. i.e.,

$$u(nTs) = u((nTs - Ts) + [\pm\Delta]$$

$$\text{or} \quad u(nTs) = u[(n-1)Ts] + b(nTs) \qquad (4)$$

The previous sample approximation $u[(n-1)T_s]$ is restored by delaying one sample period T_s .
The samples input signal $x(nT_s)$ and staircase approximated signal $\hat{x}(nT_s)$ are subtracted to get error signal $e(nT_s)$.Thus, depending on the sign of $e(nT_s)$, one bit quantizer generates an output of $+\Delta$ or $-\Delta$ .If the step size is $+\Delta$, then binary ‘1’ is transmitted and if it is $-\Delta$, then binary ‘0’ is transmitted .

Delta Modulation Receiver:

At the receiver end also known as delta demodulator, as shown in fig.2 (b) , it comprises of a low pass filter(LPF), a summer, and a delay circuit. The predictor circuit is eliminated here and hence no assumed input is given to the demodulator.

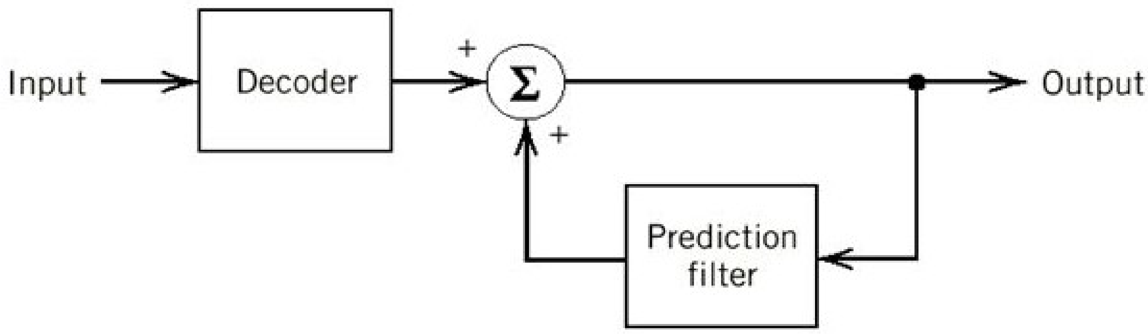


Figure 1.5 : Delta Modulation Receiver

The accumulator generates the staircase approximated signal output and is delayed by one sampling period T_s.It is then added to the input signal.If the input is binary '1' then it adds +Δ step to the previous output (which is delayed).If the input is binary '0' then one step 'Δ' is subtracted from the delayed signal. Also, the low pass filter smoothens the staircase signal to reconstruct the original message signal x(t) .

Advantages of Delta Modulation

The delta modulation has certain advantages over PCM as under :

1. Since, the delta modulation transmits only one bit for one sample, therefore the signaling rate and transmission channel bandwidth is quite small for delta modulation compared to PCM .
2. The transmitter and receiver implementation is very much simple for delta modulation. There is no analog to digital converter required in delta modulation.

Q.4. State and explain the Disadvantages of Delta Modulation

Answer:

Delta modulation have two types of quantization error:

(1) Slope overload Distortion,
(2) Granular noise Distortion.

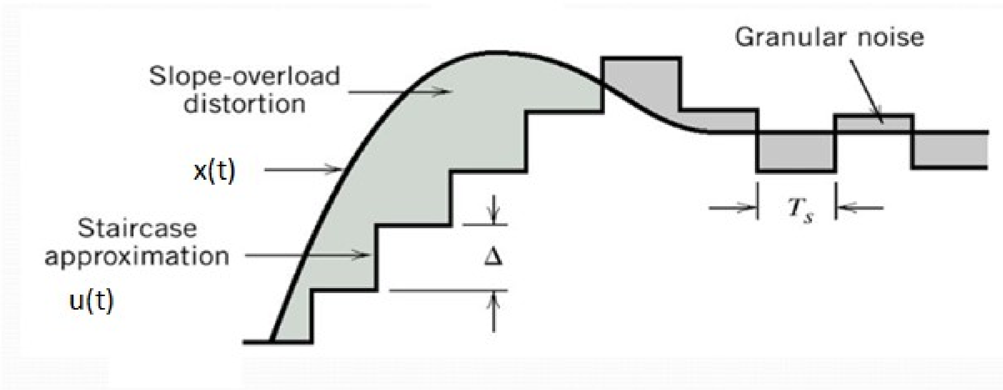


Figure1.5: Quantization errors in Delta modulation

1. Slope Overload Distortion

This distortion arises because of large dynamic range of the input signal.We can observe from fig.1 , the rate of rise of input signal x(t) is so high that the staircase signal can not approximate it, the step size 'Δ' becomes too small for staircase signal u(t) to follow the step segment of x(t).Hence, there is a large error between the staircase approximated signal and the original input signal x(t).This error or noise is known as **S**lope Overload Distortion **.**

To reduce this error, the step size must be increased when slope of signal x(t) is high.Since, the step size of delta modulator remain fixed, its maximum or minimum slopes occur along straight lines. Therefore, this modulator is known as Linear Delta Modulator (LDM) .

2. Granular Noise Distortion

Granular noise occurs when the step size is too large compared to small variation in the input signal. This means that for very small variations in the input signal, the staircase signal is changed by large amount (Δ) because of large step size.Fig.1 shows that when the input signal is almost flat , the staircase signal u(t) keeps on oscillating by ±Δ around the signal. The error between the input and approximated signal is called Granular Noise Distortion**.** The solution to this problem is to make the step size small .

Q.5 *Explain transmitter & Receiver section for Adaptive Delta modulation with mathematical analysis.*

Answer:

Adaptive Delta Modulation(ADM):

In order to overcome the quantization errors due to slope overload and granular noise, the step size (Δ) is made adaptive to variations in the input signal x(t).Particularly in the steep segment of the signal x(t), the step size is increased. And the step is decreased when the input is varying slowly. This method is known as Adaptive Delta Modulation (ADM).

ADM Transmitter:

Figure 1.5 shows the transmitter of an ADM. The logic for step size control is added in the diagram. The step size increases or decreases according to a specified rule depending on one bit quantizer output. For an example, if one bit quantizer output is high (i.e., 1), then step size may be doubled for next sample. If one bit quantizer output is low, then step size may be reduced by one step.Fig.2 shows the staircase waveforms of adaptive delta modulator and sequence of bits to be transmitted.

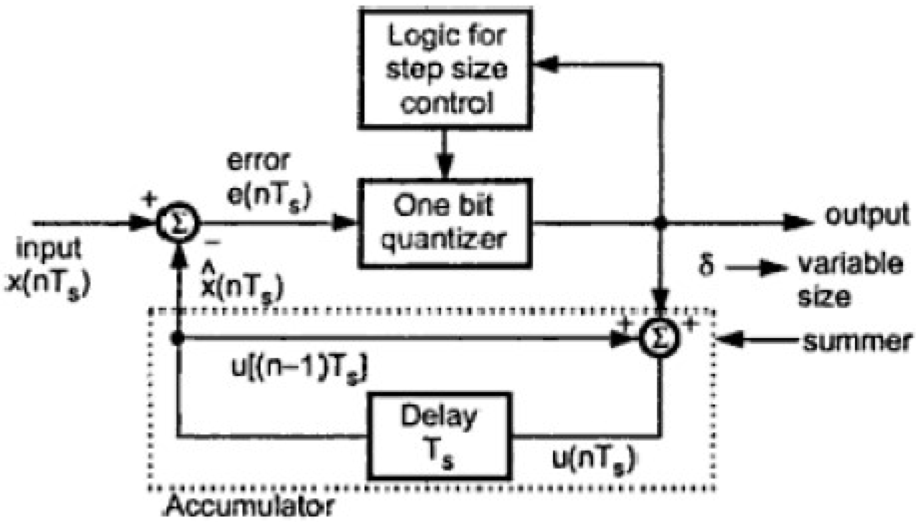


Figure1.5: ADM Transmitter

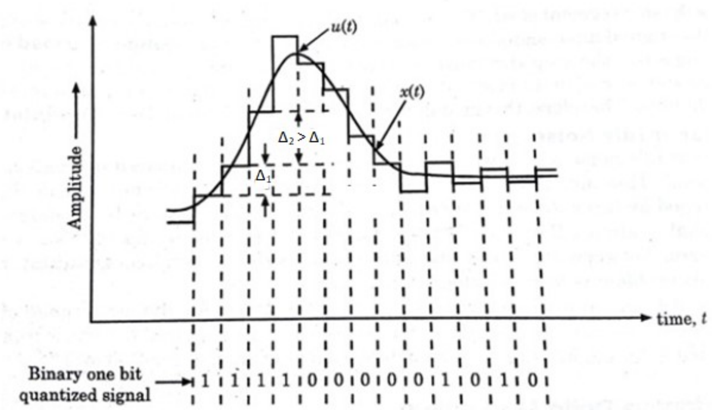


Figure1.6: ADM waveform

ADM Receiver:

Figure 1.7 shows the receiver of an ADM.

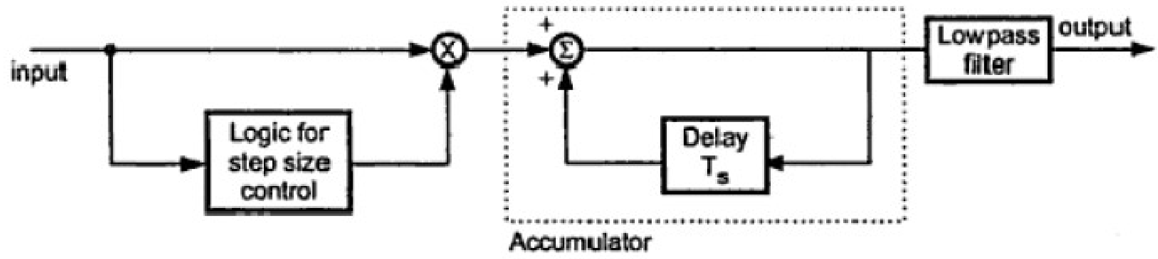


Figure1.7: ADM Receiver

The receiver has two portions. The first portion produces the step size from each incoming bit. Exactly the same process is followed as that in transmitter. The previous input and present input decide the step size. It is then applied to the second portion i.e., an accumulator which builds up staircase waveform.The low pass filter then smoothens out the staircase waveform to reconstruct the original signal.

Advantages of Adaptive Delta Modulation

Adaptive delta modulation has certain advantages over delta modulation as under :

1. The signal to noise ratio of ADM is better than that of DM because of the reduction in slope overload distortion and idle noise.
2. Because of the variable step size , the dynamic range of ADM is wider than DM.
3. Utilization of bandwidth is better in ADM than DM.

Q6. *Derive the expression for signal to quantization noise for PCM systems. Hence Prove that, (S/N)db = (4.8+6v)db.*

Answer:

Quantization Noise in PCM System:
Errors are introduced in the signal because of the quantization process. This error is called "quantization error". We define the quantization error as:

$$\varepsilon = Xq(nTs) - X(nTS) \qquad (1)$$

Let an input signal $X(nT_s)$ have an amplitude in the range of x_{max} to - x_{max}

The total amplitude range is :

$$\text{Total amplitude} = x_{max} - x_{max}$$
$$= 2x \qquad (2)$$

If the amplitude range is divided into 'q' levels of quantizer, then the step size 'Δ'.

$$\Delta = \frac{2x_{max}}{q} \qquad (3)$$

If the minimum and maximum values are equal to 1, x_{max},=1, - x_{max}=-1, then the equation (3)will be:

$$\Delta = \frac{2}{q} \qquad (4)$$

If Δ is small it can be assumed that the quantization error is uniformly distributed. The quantization noise is uniformly distributed in the interval [-Δ/2, Δ/2]. The figure.(1) shows the uniform distribution of quantization noise:

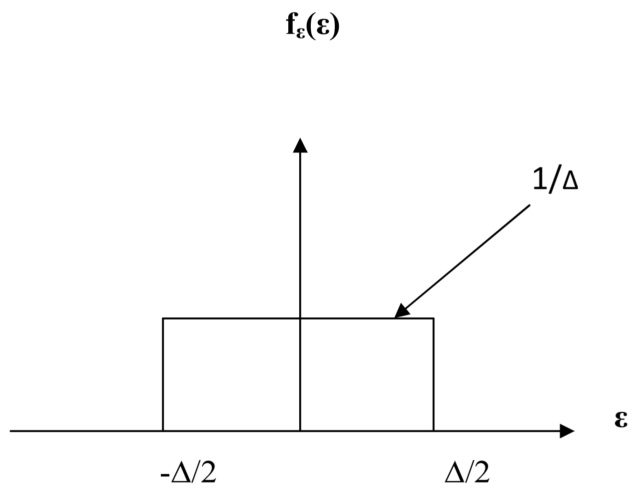


Figure1.8: The uniform distribution of quantization error

The Quantization noise power is given by:

$$\text{Quantization Noice Power} = \frac{V^2_{\text{Noise}}}{R} \qquad (5)$$

$V^2_{Noise:}$ the mean square value of noise voltage, since noise is defined by random variable "ε" and PDF $f_\varepsilon(\varepsilon)$, it's mean square value is given by :

$$V_{Noise}^2 = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \varepsilon^2 f_\varepsilon(\varepsilon).d\varepsilon \qquad (6)$$

Substitute the value of $f_\varepsilon(\varepsilon) = \frac{1}{\Delta}$ in equation(6):

$$V_{Noise}^2 = \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \varepsilon^2 \left(\frac{1}{\Delta}\right).d\varepsilon$$

$$= \frac{1}{3\Delta}\left[\frac{\Delta^3}{8} + \frac{\Delta^3}{8}\right]$$

$$= \frac{\Delta^2}{12} \qquad (7)$$

If R=1 then equation 2 is represented as

$$\text{Quantization Noise Power} = \frac{\Delta^2}{12} \qquad (8)$$

Signal to quantization Noise Ratio in PCM:

The signal to quantization noise ratio is given as:

$$\frac{S}{N} = \frac{\text{Normalized Signal Power}}{\text{Normalized Noise Power}} \qquad (9)$$

$$= \frac{\text{Normalized Signal power}}{\frac{\Delta^2}{12}} \qquad (10)$$

The number of quantization value is equal to:

$$q = 2^v$$

Putting this value in equation(3), we get:

$$\Delta = \frac{2X_{max}}{2^v}$$

Substitute this value in equation (10), we get

$$\frac{S}{N} = \frac{\text{Normalized Signal Power}}{\left[\frac{2X_{max}}{2^v}\right]^2 * \frac{1}{12}}$$

Let the normalized signal power is equal to P then the signal to quantization noise ratio will be given by:

$$\frac{S}{N} = \frac{P}{\frac{4X_{max}^2}{2^{2v}} * \frac{1}{12}} = \frac{3P * 2^{2v}}{X_{max}^2} \qquad (11)$$

If we assume that input x(t) is normalized i.e.

$$X_{max} = 1 \qquad (12)$$

Then the signal to quantization noise ratio will be given by:

$$\frac{S}{N} = 3 * 2^{2v} * P \qquad (13)$$

If Signal Power P is normalized then P≤ 1 put in equation (13).

$$\frac{S}{N} = 3 * 2^{2v} \qquad (14)$$

Expressing the signal to quantization noise in decibels as

$$\left(\frac{S}{N}\right) dB = 10 * \log_{10}\left(\frac{S}{N}\right) dB \qquad (15)$$

$$\leq 10 * \log_{10}[3 * 2^{2v}]$$

$$\leq (4.8 + 6v) dB$$

Thus the signal to quantization noise in decibels for normalized signal power is given by

$$\frac{S}{N} \leq (4.8 + 6v) dB \qquad (16)$$

Q6. *The bandwidth of signal input to the PCM is restricted to 4 kHz. The input varies from -3.8V to +3.8V and has the average power of 30mW.The required signal to noise ratio is 20 dB. The modulator produces binary output. Assume uniform quantization.*

i) Calculate the number of bits required per sample.

ii) Outputs of 30 such coders are time multiplexed .What is the minimum required transmission bandwidth for the multiplexed signal?

Answer:

The given value of signal to noise ratio is 20dB.

$$\left(\frac{S}{N}\right) dB = 10 \log_{10}\left(\frac{S}{N}\right) = 20dB$$

$$\frac{S}{N} = 100$$

i)The signal to quantization noise ratio will be given by:

$$\frac{S}{N} = \frac{3P * 2^{2v}}{X_{max}^2}$$

The given data is Xmax=3.8V,P=30mW,S/N=100

$$\therefore \qquad 100 = \frac{3 * 30 * 10^{-3} * 2^{2v}}{(3.8)^2}$$

$v = 6.98 \approx 7 bits$

ii)The maximum frequency is

$W = 4kHz$

The transmission bandwidth is given by the equation

$B_T \geq vW$

There are 30 such coders are time multiplexed. Therefore the transmission bandwidth is given as

$B_T \geq 30 * v * W$

$\geq 30 * 7 * 4kHz$

$\geq 840kHz$

The signaling rate r is given as

$r = 2 * B_T$

$\therefore \quad r = 2 * 840kHz$

$r = 1680\ bits/Sec$

Q7. *A Television signal with a bandwidth of 4.2 MHz is transmitted using binary PCM. The number of quantization levels are 512.*

Calculate :—

(i) Code word length.

(ii) Transmission bandwidth.

(iii) Final bit rate.

(iv) Output signal to quantization noise ratio.

Answer:

Given data:W=4.2MHz and Q=512

i)Number of bits and quantization levels are related by the equation given below:

$q = 2^v$

$512 = 2^v$

$\log 512 = v \log 2$

$$v = \frac{\log 512}{\log 2}$$

$$v = 9\ bits$$

ii)The transmission channel bandwidth is given as:

$$B_T \geq v * W$$

$$\geq 9 * 4.2 * 10^6 Hz$$

$$\geq 38.7 MHz$$

iii)The signalling rate is given as:

$$r = v * f_s$$

Sampling frequency is given as:

$$f_s \geq 2W$$

$$f_s \geq 2 * 4.2 * 10^6$$

$$\geq 8.4 \text{ MHz}$$

$$r = 9 * 8.4 * 10^6$$

$$r = 75.6 * 10^6\ bits/sec$$

The transmission bandwidth is given as:

$$B_T \geq \frac{1}{2} * r$$

$$B_T \geq \frac{1}{2} * 75.4 * 10^6$$

$$B_T \geq 37.8\ MHz$$

iv)The signal to noise ratio is given as:

$$\left(\frac{S}{N}\right) dB \leq (4.8 + 6 * v)dB$$

$$\leq (4.8 + 6 * 9)dB$$

$$\leq 58.8\ dB$$

Q.8 *A signal of bandwidth 3.5kHz is sampled quantized and coded by a PCM system. The coded signal is then transmitted over a transmission channel of supporting a transmission rate of 50k bits/sec.Calculate the maximum signal to noise ratio that can be obtained by this system.The input signal has peak to peak value of 4volts and rms value of 0.2V.*

Answer:

The maximum frequency of the signal is given as

$$W = 3.5kHz$$

Therefore sampling frequency will be

$$f_s \geq 2 * W$$

$$\geq 2 * 3.5kHz$$

$$\geq 7kHz$$

The signaling rate is given by the equation

$$r = vf_s$$

Put the values r=50 * 10^3 bits/sec and $f_s \geq 7 * 10^3$

$$\therefore \quad 50 * 10^3 \leq v * 7 * 10^3$$

$$v \geq 7.142 \approx 8\ bits$$

The rms value of the signal is given as 0.2V. Therefore the Normalized signal power will be given as

$$Normalized\ signal\ power = \frac{(0.2)^2}{1} \qquad [as\ R = 1\ for\ normalized\ power]$$

$$\therefore P = 0.04W$$

The maximum signal to noise ratio is given by

$$\frac{S}{N} = \frac{3P * 2^{2v}}{X_{max}^2}$$

Put P=0.4W, v=8 bits, x_{max}=2 V in the above equation:

$$\frac{S}{N} = \frac{3 * (0.04) * 2^{2*8}}{2^2}$$

$$= 1966.08 = 33db$$

***Q9**.Derive the expression for signal to quantization noise for PCM systems that employs linear quantization technique Assume that input to PCM system is a sinusoidal signal.*

Answer:

Assume that the modulating signal be a sinusoidal voltage having peak amplitude A_m.

The power of the signal is expressed as:

$$P = \frac{V^2 rms}{R}$$

$$P = \frac{\frac{A_m}{\sqrt{2}}}{R}$$

Where $\frac{A_m}{\sqrt{2}}$ is an rms value. Put R=1 in above equation as P is normalized.

$$\therefore P = \frac{A_m}{\sqrt{2}} \qquad (1)$$

Signal to quantization noise ratio is given as:

$$\frac{S}{N} = \frac{3P * 2^{2v}}{X_{max}^2} \qquad (2)$$

Put $P = \frac{A_m}{\sqrt{2}}$ and $X_{max}=A_m$ in the equation 2.

$$\frac{S}{N} = \frac{3\frac{A_m}{\sqrt{2}} * 2^{2v}}{A_m^2} \qquad (3)$$

Expressing Signal to quantization noise ratio in dB:

$$\left(\frac{S}{N}\right) dB = 10 * \log_{10}\left(\frac{S}{N}\right) dB \qquad (4)$$

$$\left(\frac{S}{N}\right) dB \leq 10 * \log_{10}[1.5 * 2^{2v}]$$

$$\left(\frac{S}{N}\right) dB \leq (1.8 + 6v) dB \qquad (5)$$

Thus for sinusoidal wave (S/N)db is given as:

$$\left(\frac{S}{N}\right) dB \leq (1.8 + 6v) dB$$

Q10*.The information is an analog signal voltage waveform is to be transmitted over a PCM system with an accuracy of k 0.1% (full scale). The analog voltage wave form has a bandwidth of 100 MHz and an amplitude range of – 10 to 10 volts.*

(i) Determine the maximum sampling rate required.

(ii) Determine the number of bits rate in each PCM word.

(iii) Determine the minimum bit rate required in PCM signal.

(iv) Determine the minimum absolute channel bandwidth required for transmission of PCM signal.

Answer:

The accuracy given is ±0.1 %

i.e the maximum quantization error is ± 0.1%

$\therefore\ \varepsilon_{max} = \pm 0.1\% = 0.001$

The maximum quantization error for an uniform quantizer is given as

$$\varepsilon_{max} = \left|\frac{\Delta}{2}\right|$$

$$\left|\frac{\Delta}{2}\right| = 0.001$$

$$i.e. \quad step\ size\ \Delta = 0.002$$

Step size can be also expressed as:

$$\Delta = \frac{2x_{max}}{q}$$

$$0.002 = \frac{2 * 10}{q}$$

$$q = \frac{0.002}{20}$$

$$q = 10{,}000$$

$$\therefore Number\ of\ quantization\ levels = 10{,}000$$

a)Maximum frequency is given as:

$$W = 100Hz$$

Them Sampling frequency is given as:

$$f_s \geq 2 * W \geq 2 * 100 \geq 200Hz$$

b)The number of bits per sample can be calculated as:

$$q = 2^v$$

$$10000 = 2^v$$

$$log_{10}10000 = v\,log_{10}2$$

$$v = \frac{\log 10000}{log 2}$$

$$v = 13.288$$

$$v = 13\ bits$$

c) Signalling rate is given as:

$$r = v * f_s$$

$$= 13 * 200$$

$$= 2600\ bits\ /sec$$

d) The transmission channel for PCM is given as:

$$B_T = \frac{1}{2} * r \qquad = \frac{1}{2} * 2600$$

$$= 1300\ Hz$$

UNIT No II

Q1: *Analog signal band limited to 4 KHz is quantized in 8 levels of PCM with probability given below. Find the total amount of information in PCM signal.*
Pk=1/4,1/5,1/5,1/10,1/10,1/20,1/20,1/20

Answer:

k=1,2,3,4,5,6,7,8

Pk=1/4,1/5,1/5,1/10,1/10,1/20,1/20,1/20

Pk=P1,P2,P3,P4,P5,P6,P7,P8

$$I(S_k) = \log_2 \frac{1}{Pk}$$
$$I(S_1) = \log_2 \frac{1}{P1} = \log_2 \frac{1}{1/4} = 2$$

Similarly

I(S2)=2.32,I(S3)=2.32,I(S4)=3.32,I(S5)=3.32,I(S6)=4.32,I(S6)=4.32,I(S7)=4.32,I(S8)=4.32

$$I(S_k) = \sum_{k=0}^{k-1} \log_2 \frac{1}{Pk}$$

$$I(S_k) = \sum_{k=0}^{8} \log_2 \frac{1}{Pk}$$
$$I(S_k) = \log_2 \frac{1}{P0} + \log_2 \frac{1}{P1} + \log_2 \frac{1}{P2} + \log_2 \frac{1}{P3} + \log_2 \frac{1}{P4} + \log_2 \frac{1}{P5} + \log_2 \frac{1}{P6} + \log_2 \frac{1}{P7} + \log_2 \frac{1}{P8}$$
=2+2.32+2.32+3.32+3.32+4.32+4.32+4.32+4.32

=26.24

Q2: *Consider a discrete memoryless source with alphabet s0,s1,s2 and statistics 0.5,0.25,0.25*
(i) Apply the huffman algorithm to this source.
(ii) let the source be extended to order two. Apply the huffman algorithm to the resulting extended source
(iii) Compare the average code word length calculated in part (ii) with he entropy of the original source

Answer:

$$H(S)=\sum_{k=0}^{k-1} Pk.I(Sk)$$

$$H(S)=\sum_{k=0}^{2} Pk.\log_2 \frac{1}{Pk}$$

$$H(S)=P0.\log_2 \frac{1}{P0}+P1.\log_2 \frac{1}{P1}+P2.\log_2 \frac{1}{P2}$$

$$H(S)=(1/4).\log_2 \frac{1}{1/4}+(1/4).\log_2 \frac{1}{1/4}+(1/2).\log_2 \frac{1}{1/2}$$

H(S)=3/2 bits

H(S^2)=Second order extension of source

S={S0,S1,S2}

S^2={S0S0,S0S1,S0S2, S1S0,S1S1,S1S2, S2S0,S2S1,S2S2}

$$S^2=\{\sigma_0,\sigma_1,\sigma_2,\sigma_3,\sigma_4,\sigma_5,\sigma_6,\sigma_7,\sigma_8\}$$

Symbols, S^2	Corresponding Symbols	Prob. $P(\sigma_i)$ i=0 to 8	Prob. $P(\sigma_i)$ i=0 to 8
σ_0	S0S0	$P(\sigma_0) = P(S0)P(S0)$	$P(\sigma_0) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
σ_1	S0S1	$P(\sigma_1) = P(S0)P(S1)$	$P(\sigma_1) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
σ_2	S0S2	$P(\sigma_2) = P(S0)P(S2)$	$P(\sigma_2) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$
σ_3	S1S0	$P(\sigma_3) = P(S1)P(S0)$	$P(\sigma_3) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
σ_4	S1S1	$P(\sigma_4) = P(S1)P(S2)$	$P(\sigma_4) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$
σ_5	S1S2	$P(\sigma_5) = P(S1)P(S3)$	$P(\sigma_5) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$
σ_6	S2S0	$P(\sigma_6) = P(S2)P(S0)$	$P(\sigma_6) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$
σ_7	S2S1	$P(\sigma_7) = P(S2)P(S1)$	$P(\sigma_7) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$
σ_8	S2S2	$P(\sigma_8) = P(S2)P(S2)$	$P(\sigma_8) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$$H(S^2) = \sum_{k=0}^{8} P\sigma i.\log_2 \frac{1}{P\sigma i}$$

$$H(S^2) = P\sigma_0.\log_2 \frac{1}{P\sigma_0} + P\sigma_1.\log_2 \frac{1}{P\sigma_1} + P\sigma_2.\log_2 \frac{1}{P\sigma_2} + P\sigma_3.\log_2 \frac{1}{P\sigma_3} +$$
$$P\sigma_4.\log_2 \frac{1}{P\sigma_4} + P\sigma_5.\log_2 \frac{1}{P\sigma_5} + P\sigma_6.\log_2 \frac{1}{P\sigma_6} + P\sigma_7.\log_2 \frac{1}{P\sigma_7} + P\sigma_8.\log_2 \frac{1}{P\sigma_8}$$

$$H(S^2) = \frac{1}{16}.\log_2 \frac{1}{1/16} + \frac{1}{16}.\log_2 \frac{1}{1/16} + \frac{1}{8}.\log_2 \frac{1}{1/8} + \frac{1}{16}.\log_2 \frac{1}{1/16} + \frac{1}{16}.\log_2 \frac{1}{1/16} +$$
$$\frac{1}{8}.\log_2 \frac{1}{1/8} + \frac{1}{8}.\log_2 \frac{1}{1/8} + \frac{1}{8}.\log_2 \frac{1}{1/8} + \frac{1}{4}.\log_2 \frac{1}{1/4}$$

$$H(S^2) = 3bits$$

Q3:*A source emits one of 4 symbols S0,S1,S2,S3 with probabilities 1/3,1/6,1/4,1/4. The successive symbols emitted by the source are statistically independent.Calculate entropy of the source.*

Answer:

$$H(S) = \sum_{k=0}^{k-1} Pk.\log_2 \frac{1}{Pk}$$

$$H(S) = \sum_{k=0}^{3} Pk.\log_2 \frac{1}{Pk}$$

$$H(S) = P0.\log_2 \frac{1}{P0} + P1.\log_2 \frac{1}{P1} + P2.\log_2 \frac{1}{P2} + P3.\log_2 \frac{1}{P3}$$

$$H(S) = (1/3).\log_2 \frac{1}{1/3} + (1/6).\log_2 \frac{1}{1/6} + (1/4).\log_2 \frac{1}{1/4} + (1/4).\log_2 \frac{1}{1/4}$$

H(S)=1.95bits

Q4:*Consider discrete memoryless source with source alphabet S={S0,S1,S2} and source statistics={0.7,0.15,0.15}*

a)Calculate entropy of source

b)Calculate entropy of second order extension of source.

Prove that $H(S^2) = 2.H(S)$

Answer:

$$H(S) = \sum_{k=0}^{k-1} Pk.\log_2 \frac{1}{Pk}$$

$$H(S) = \sum_{k=0}^{2} Pk.\log_2 \frac{1}{Pk}$$

$$H(S) = P0.\log_2 \frac{1}{P0} + P1.\log_2 \frac{1}{P1} + P2.\log_2 \frac{1}{P2}$$

$$H(S) = (0.7).\log_2 \frac{1}{0.7} + (0.15).\log_2 \frac{1}{0.15} + (0.15).\log_2 \frac{1}{0.15}$$

H(S)=1.17

$H(S^2)$=Second order extension of source

S={S0,S1,S2}

S^2={S0S0,S0S1,S0S2, S1S0,S1S1,S1S2, S2S0,S2S1,S2S2}

$S^2 = \{\sigma_0, \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8\}$

Symbols, S^2	Corresponding Symbols	Prob. $P(\sigma_i)$ i=0 to 8	Prob. $P(\sigma_i)$ i=0 to 8
σ_0	S0S0	$P(\sigma_0) = P(S0)P(S0)$	$P(\sigma_0) = 0.7 \times 0.7 = 0.4$
σ_1	S0S1	$P(\sigma_1) = P(S0)P(S1)$	$P(\sigma_1) = 0.7 \times 0.15 = 0.105$
σ_2	S0S2	$P(\sigma_2) = P(S0)P(S2)$	$P(\sigma_2) = 0.7 \times 0.15 = 0.105$
σ_3	S1S0	$P(\sigma_3) = P(S1)P(S0)$	$P(\sigma_3) = 0.15 \times 0.7 = 0.105$
σ_4	S1S1	$P(\sigma_4) = P(S1)P(S2)$	$P(\sigma_4) = 0.15 \times 0.15 = 0.0225$
σ_5	S1S2	$P(\sigma_5) = P(S1)P(S3)$	$P(\sigma_5) = 0.15 \times 0.15 = 0.0225$
σ_6	S2S0	$P(\sigma_6) = P(S2)P(S0)$	$P(\sigma_6) = 0.15 \times 0.7 = 0.105$
σ_7	S2S1	$P(\sigma_7) = P(S2)P(S1)$	$P(\sigma_7) = 0.15 \times 0.15 = 0.0225$
σ_8	S2S2	$P(\sigma_8) = P(S2)P(S2)$	$P(\sigma_8) = 0.15 \times 0.15 = 0.0225$

$$H(S^2) = \sum_{k=0}^{8} P\sigma i.\log_2 \frac{1}{P\sigma i}$$

$$H(S^2)=P\sigma_0.\log_2\frac{1}{P\sigma_0}+P\sigma_1.\log_2\frac{1}{P\sigma_1}+P\sigma_2.\log_2\frac{1}{P\sigma_2}+P\sigma_3.\log_2\frac{1}{P\sigma_3}+P\sigma_4.\log_2\frac{1}{P\sigma_4}+P\sigma_5.\log_2\frac{1}{P\sigma_5}+P\sigma_6.\log_2\frac{1}{P\sigma_6}+P\sigma_7.\log_2\frac{1}{P\sigma_7}+P\sigma_8.\log_2\frac{1}{P\sigma_8}$$

$$H(S^2)=0.4.\log_2\frac{1}{0.4}+0.105.\log_2\frac{1}{0.105}+0.105.\log_2\frac{1}{0.105}+0.105.\log_2\frac{1}{0.105}+0.0225.\log_2\frac{1}{0.0225}+0.0225.\log_2\frac{1}{0.0225}+0.105.\log_2\frac{1}{0.105}+0.0225.\log_2\frac{1}{0.0225}+0.0225.\log_2\frac{1}{0.0225}$$

$H(S^2)$=2.38 bits

$H(S^2)$=2H(S)

Hence proved

Q5:*Find Lempel Ziv(LZ) Source code for the Binary source Sequence given below sequence:0001011100101001 01. Assume 0 and 1 are already present in the code.*

Answer:

$$\underbrace{0\quad 0}\quad \underbrace{0\quad 1}\quad \underbrace{0\quad 1\quad 1}\quad \underbrace{1\quad 0}\quad \underbrace{0\quad 1\quad 0}\quad \underbrace{1\quad 0\quad 0}\quad \underbrace{1\quad 0\quad 1}$$

Numerical position	1	2	3	4	5	6	7	8	9
Subsequences	0	1	00	01	011	10	010	100	101
Numerical representation			11	12	42	21	41	61	62
Binary encoded blocks			0010	0011	1001	0100	1000	1100	1101

Q6*: Find Lempel Ziv(LZ) Source code for the Binary source Sequence given below Sequence:111010011000101 Assume 0 and 1 are already present in the code.*

Answer:

1 1 1 0 1 0 0 1 1 0 0 0 1 0 1

Numerical position	1	2	3	4	5	6	7	8
Subsequences	0	1	11	10	100	110	00	101
Numerical representation			22	21	41	31	11	42
Binary encoded blocks			0101	0100	1000	0110	0010	1001

Q7: *Find Lempel Ziv(LZ) Source code for the Binary source Sequence given below Sequence:000101110010100101*

Answer:

0 0 0 1 0 1 1 1 0 0 1 0 1 0 0 1 0 1

Numerical position	1	2	3	4	5	6	7	8	9
Subsequences	0	1	00	01	011	10	010	100	101
Numerical representation			11	12	42	21	41	61	62
Binary encoded blocks			0010	0011	1001	0100	1000	1100	1101

Q8: *Find Lempel Ziv(LZ) Source code for the Binary source Sequence given below*
Sequence:1110100110001011010

Answer:

$$\underbrace{1\ \ 1}\ \ \underbrace{1\ \ 0}\ \ \underbrace{1\ \ 0\ \ 0}\ \ \underbrace{1\ \ 1\ \ 0}\ \ \underbrace{0\ \ 0}\ \ \underbrace{1\ \ 0\ \ 1}\ \ \underbrace{1\ \ 0\ \ 1\ \ 0}$$

Numerical position	1	2	3	4	5	6	7	8	9
Subsequences	0	1	11	10	100	110	00	101	1010
Numerical representation			22	21	41	31	11	42	81
Binary encoded blocks			00101	00100	01000	00110	00010	01001	10000

Q9: *Encode the sequence using Lampel Ziv coding for 1110100110001011010*
Assume 0 and 1 are already present in the code.

Answer:

Numerical position	Subsequences	Numerical Representation	Binary encoded block
1	0	-	
2	1	-	
3	11	22	00101
4	10	21	00100
5	100	41	01000
6	110	31	00110
7	00	11	00010
8	101	42	01001
9	1010	81	10000

Q10: *Find Lempel Ziv Source code for the Binary source Sequence given below 000100100000001100001000000010100001000000110100000001 1*

Answer:

Numerical position	Subsequences	Numerical Representation	Binary encoded block
1	0	-	
2	1	-	
3	00	11	00010
4	01	12	00011
5	001	32	00111
6	000	31	00110
7	0001	62	01101
8	10	21	00100
9	00010	71	01110
10	0000	61	01100
11	0010	51	01010
12	100	81	10000
13	00100	111	10110
14	00001	102	10101
15	101	82	10001
16	00000	101	10100
17	0011	52	01011

Q11: *Find Lempel Ziv Source code for the Binary source Sequence given below 0001001000000011000010000000100000010100001000000110100000001100…*

Answer:

Numerical position	Subsequences	Numerical Representation	Binary encoded block
1	0	-	
2	1	-	
3	00	11	000010
4	01	12	000011
5	001	32	000111
6	000	31	000110
7	0001	62	001101
8	10	21	000100
9	00010	71	001110
10	0000	61	001100
11	000100	91	010010
12	00001	102	010101
13	010	41	001000
14	000100	111	010110

15	00011	72	001111
16	0100	131	011010
17	00000	101	010100
18	11	22	000101
19	000000	171	100010

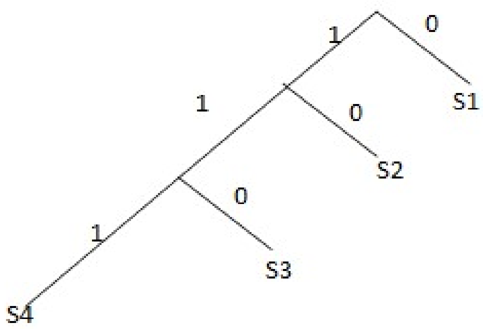
Q12*: Explain Prefix code and also identify the prefix code out of listed below. Construct their individual decision tree*

Answer:

Prefix Code is defined as a code in which no codeword is the prefix of any other code word.

Source symbols	Probability of Occurance	Code I	Code II	Code III
S0	0.45	0	0	0
S1	0.1875	1	101	01
S2	0.1875	00	110	011
S3	0.125	11	111	0111

Code II

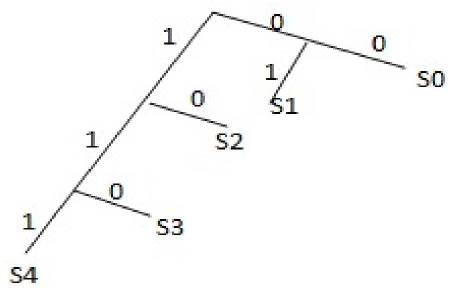


Q13: Identify the prefix code out of listed below.Construct their individual decision tree

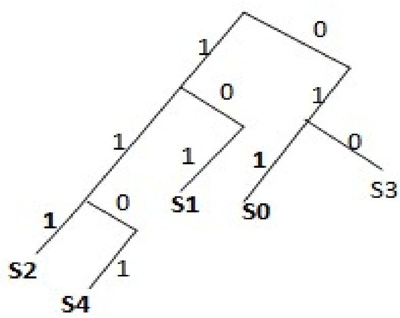
Source Symbols	Prob. Of Occurance	Code I	Code II	CodeIII	CodeIV	CodeV	CodeVI
S0	0.35	0	0	00	0	00	011
S1	0.275	101	10	01	01	0011	101
S2	0.275	110	110	10	011	0101	111
S3	0.05	11110	111	110	110	01011	010
S4	0.05	1111	11	111	111	01100	1101

Answer:

code III



Code IV

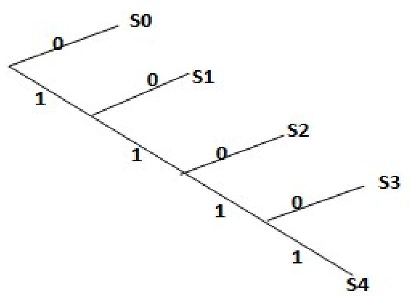


Q14: *Identify the prefix code out of listed below. Construct their individual decision tree*

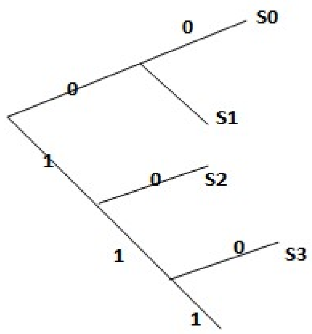
Source Symbols	Code I	Code II	CodeIII	CodeIV
S0	0	0	0	00
S1	10	01	01	01
S2	110	001	011	10
S3	1110	0010	110	110
S4	1111	0011	111	111

Answer:

code I



Code IV



Q15: *The five symbols of the alphabet of a DMS & their probabilities are shown in table below:*

Symbol	Probability
S0	0.4
S1	0.2
S2	0.2
S3	0.1
S4	0.1

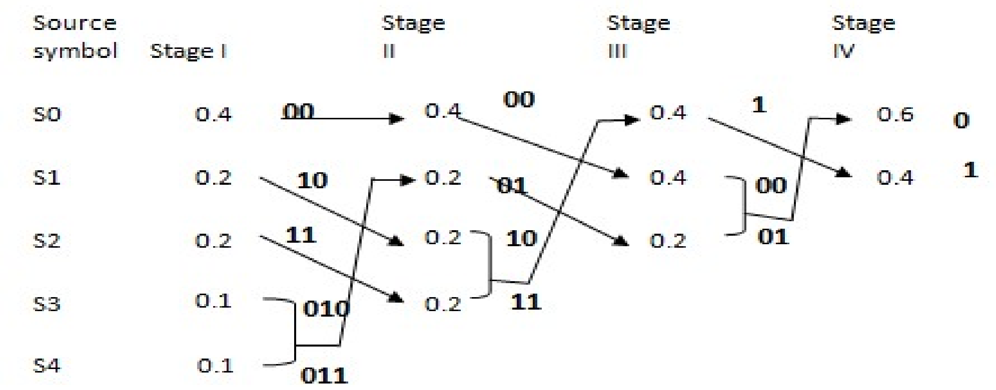
Applying Huffman coding by both the methods as high as possible Calculate-

a) Avg. code word length.

b) Entropy of the source.

c) Efficiency of the source.

Answer:



Arranged as highest to lowest values then last two values are added then compare with the highest value and arrange. And so on up to only two values left.

Apply the logic to last two values as 0 and 1 and according to the direction of arrow assign the values.

Symbol	Prob	Codeword	Length of codeword
S0	0.4	00	2 bit
S1	0.2	10	2 bit
S2	0.2	11	2 bit
S3	0.1	010	3bit
S4	0.1	011	3 bit

b)Average code word length

$$\overline{L} = \sum_{k=0}^{k-1} P_k.l_k = \sum_{k=0}^{4} P_k.l_k$$

$$\overline{L} = P_0.l_0 + P_1.l_1 + P_2.l_2 + P_3.l_3 + P_4.l_4$$

$$\overline{L} = (0.4 \times 2) + (0.2 \times 2) + (0.2 \times 3) + (0.1 \times 3) + (0.1 \times 3)$$

$$\overline{L} = 2.2$$

Entropy

$$H(S) = \sum_{k=0}^{k-1} Pk.\log_2 \frac{1}{Pk}$$

$$H(S) = \sum_{k=0}^{4} Pk.\log_2 \frac{1}{Pk}$$

$$H(S) = P0.\log_2 \frac{1}{P0} + P1.\log_2 \frac{1}{P1} + P2.\log_2 \frac{1}{P2} + P3.\log_2 \frac{1}{P3} + P4.\log_2 \frac{1}{P4}$$

$$H(S) = 0.4\log_2 \frac{1}{0.4} + 0.2\log_2 \frac{1}{0.2} + 0.2\log_2 \frac{1}{0.2} + 0.1\log_2 \frac{1}{0.1} + 0.1\log_2 \frac{1}{0.1}$$

$$H(S) = 0.428 + 0.464 + 0.464 + 0.33 + 0.33$$

$$H(S) = 2.12 bits / symbol$$

d)Variance

$$\sigma^2 = \sum_{k=0}^{k-1} P_k (l_k - \overline{L})^2$$

$$\sigma^2 = \sum_{k=0}^{4} P_k (l_k - \overline{L})^2$$

$$\sigma^2 = P_0(l_0 - \overline{L})^2 + P_1(l_1 - \overline{L})^2 + P_2(l_2 - \overline{L})^2 + P_3(l_3 - \overline{L})^2 + P_4(l_4 - \overline{L})^2$$

$$\sigma^2 = [0.4(2-2.2)^2] + [0.2(2-2.2)^2] + [0.2(2-2.2)^2] + [0.1(3-2.2)^2] + [0.1(3-2.2)^2]$$

$$\sigma^2 = 0.16$$

e)Percentage increase in Lavg $= \dfrac{\overline{L}avg - H(S)}{H(S)} \times 100$

$$= \frac{2.2 - 2.12}{2.12} \times 100$$

% change in Lavg=3.77%

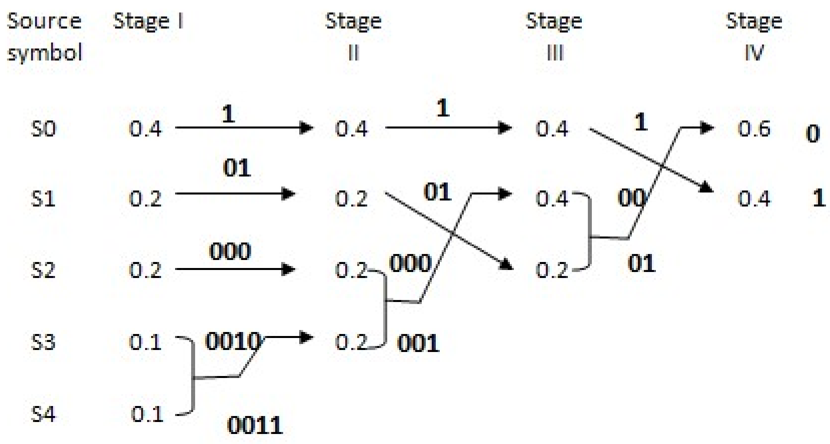
Q16:*The five symbols of the alphabet of a DMS & their probabilities are shown in table below:*

Symbol	Probability
S0	0.4
S1	0.2
S2	0.2
S3	0.1
S4	0.1

Applying Huffman coding by both the methods as low as possible.
Calculate-
a) Create Huffman tree.
b) Avg. code word length.
c) Entropy of the source.
d) Efficiency of the source.
e)Variance

Answer:

Arrange prob as high as possible then in stages aarrange prob. As low as possible and assign them 0 and 1 values



Symbol	Prob	Codeword	Length of codeword
S0	0.4	1	1 bit
S1	0.2	01	2 bit
S2	0.2	000	3 bit
S3	0.1	0010	4 bit
S4	0.1	0011	4 bit

b)Average code word length

$$\overline{L} = \sum_{k=0}^{k-1} P_k.l_k = \sum_{k=0}^{4} P_k.l_k$$

$$\overline{L} = P_0.l_0 + P_1.l_1 + P_2.l_2 + P_3.l_3 + P_4.l_4$$

$$\overline{L} = (0.4\times 1) + (0.2\times 2) + (0.2\times 3) + (0.1\times 4) + (0.1\times 4)$$
$$\overline{L} = 2.2$$

c)Entropy

$$H(S) = \sum_{k=0}^{k-1} Pk.\log_2 \frac{1}{Pk}$$

$$H(S) = \sum_{k=0}^{4} Pk.\log_2 \frac{1}{Pk}$$

$$H(S) = P0.\log_2 \frac{1}{P0} + P1.\log_2 \frac{1}{P1} + P2.\log_2 \frac{1}{P2} + P3.\log_2 \frac{1}{P3} + P4.\log_2 \frac{1}{P4}$$

$$H(S) = 0.4\log_2 \frac{1}{0.4} + 0.2\log_2 \frac{1}{0.2} + 0.2\log_2 \frac{1}{0.2} + 0.1\log_2 \frac{1}{0.1} + 0.1\log_2 \frac{1}{0.1}$$

$$H(S) = 0.428 + 0.464 + 0.464 + 0.33 + 0.33$$
$$H(S) = 2.12 bits/symbol$$

d)Variance

$$\sigma^2 = \sum_{k=0}^{k-1} P_k (l_k - \overline{L})^2$$

$$\sigma^2 = \sum_{k=0}^{4} P_k (l_k - \overline{L})^2$$

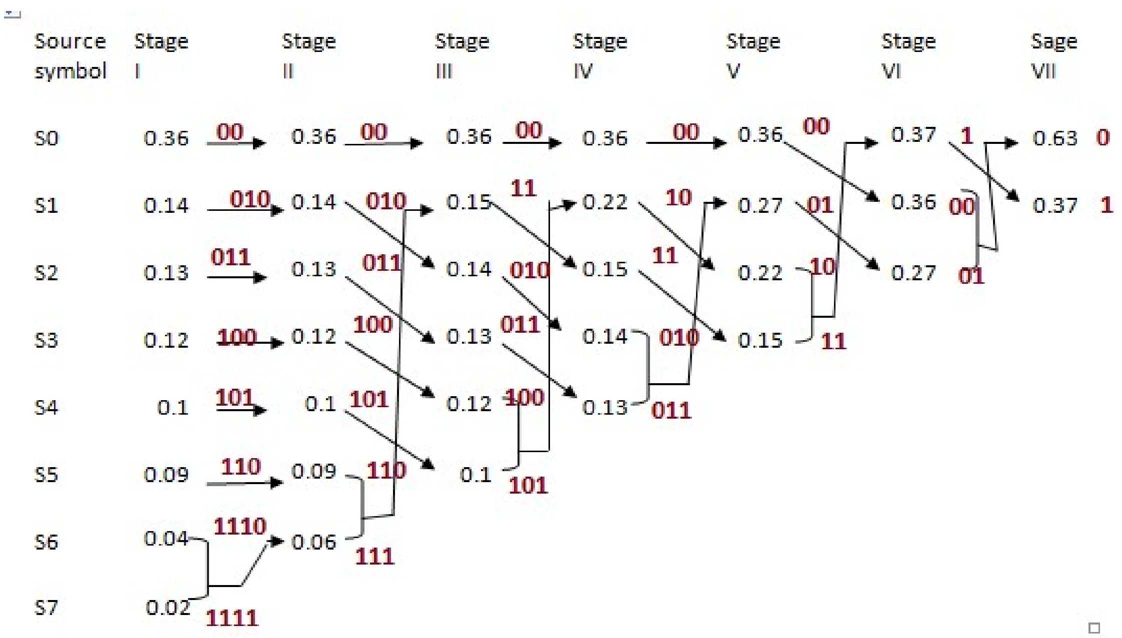
$$\sigma^2 = P_0(l_0-\overline{L})^2 + P_1(l_1-\overline{L})^2 + P_2(l_2-\overline{L})^2 + P_3(l_3-\overline{L})^2 + P_4(l_4-\overline{L})^2$$

$$\sigma^2 = [0.4(1-2.2)^2] + [0.2(2-2.2)^2] + [0.2(3-2.2)^2] + [0.1(4-2.2)^2] + [0.1(4-2.2)^2]$$

$$\sigma^2 = 0.576 + 0.008 + 0.126 + 0.324 + 0.324$$

$$\sigma^2 = 1.36$$

Q17:*Determine Huffman code for the discrete memoryless source,moving a combined symbol as high as possible .Calculate average codeword length,entropy,variance of average codeword length.*
Prob.:0.36,0.14,0.13,0.12,0.10,0.09,0.04,0.02
Answer:



Symbol	Prob	Codeword	Length of codeword
S0	0.36	00	2 bit
S1	0.14	010	3 bit
S2	0.13	011	3 bit
S3	0.12	100	3 bit
S4	0.10	101	3 bit
S5	0.09	110	3 bit
S6	0.04	1110	4 bit
S7	0.02	1111	4 bit

Average codeword length

$$\overline{L} = \sum_{k=0}^{k-1} P_k . l_k = \sum_{k=0}^{7} P_k . l_k$$

$$\overline{L} = P_0.l_0 + P_1.l_1 + P_2.l_2 + P_3.l_3 + P_4.l_4 + P_5.l_5 + P_6.l_6 + P_7.l_7$$

$$\overline{L} = (0.36\times 2) + (0.14\times 3) + (0.13\times 3) + (0.12\times 3) + (0.10\times 3) + (0.09\times 3) + (0.04\times 4) + (0.02\times 4)$$

$$\overline{L} = 2.7 bits / symbol$$

Entropy

$$H(S)=\sum_{k=0}^{k-1} Pk.\log_2 \frac{1}{Pk}$$

$$H(S)=\sum_{k=0}^{7} Pk.\log_2 \frac{1}{Pk}$$

H(S)=0.53+0.39+0.38+0.36+0.33+0.32+0.18+0.11

H(S)=2.62 bits/symbol

Variance

$$\sigma^2=\sum_{k=0}^{k-1} P_k(l_k-\overline{L})^2$$

$$\sigma^2=\sum_{k=0}^{7} P_k(l_k-\overline{L})^2$$

$$\sigma^2=[0.36(2-2.7)^2]+[0.14(3-2.7)^2]+[0.13(3-2.7)^2]+[0.12(3-2.7)^2]+$$
$$[0.10(3-2.7)^2]+[0.09(3-2.7)^2]+[0.04(4-2.7)^2]+[0.02(4-2.7)^2]$$

$$\sigma^2=0.176+0.0126+0.0117+0.0108+0.009+0.0081+0.0676+0.0338$$
$$\sigma^2=0.3296$$

Efficiency

$$\%\eta=\frac{H(S)}{Lavg}\times 100$$

UNIT No III

Q1. *Draw and explain the scheme*
(i) for generating the signal Si(t) and
(ii) for generating the set of coefficients of Signal vector {Si}

Answer:

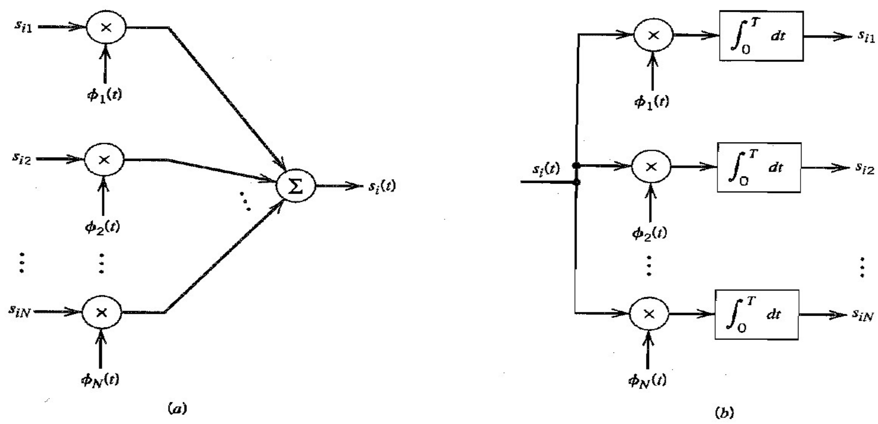


Fig (a) synthesizer for generating the signal Si(t)

(b) Analysis for generating the set of signal vectors{si}

- Let the task of transforming an incoming message mi,i=1,2,3....M into a modulated wave Si(t) may be divided into separate discrete time and continuous time operations.
- The justification for this separation lies in the Gram-Schmidt orthogonalization procedure which permits the representation of any set of M energy signals,{Si(t)} as linear combination of N orthonormal basis functions. Where N≤M i.e. represent the given set of real valued energy signals S1(t),S2(t),....SM(t) each of duration T sec. In the form
- $$S_i(t) = \sum_{j=1}^{N} S_{ij}\Phi_j(t) \qquad \begin{matrix} 0 \le t \le T \\ i = 1,2,......,M \end{matrix}$$
- Where the coefficient of expansion Sij are defined by

$$S_{ij} = \int_0^T S_i(t)\Phi_j(t)dt \qquad \begin{matrix} i = 1,2,......,M \\ j = 1,2,......,N \end{matrix}$$

- The real valued basis functions $\Phi_1(t), \Phi_2(t)..........\Phi_N(t)$ are orthonormal
- $\int_0^T \Phi_i(t)\Phi_j(t)dt = \begin{cases} 1 \quad \text{if} \quad \text{i=j,normal signal} \\ 0 \quad \text{if} \quad i \neq j \text{ ,orthogonal signal,perpendicular} \end{cases}$
- If the two signal are equal then it is normal signal
- If the two signals are not equal then it is orthogonal signal
- The first condition states that each basis function is normalized to have unit energy
- The second condition states that each basis function $\Phi_1(t), \Phi_2(t)..........\Phi_N(t)$ are orthogonal w.r.t. each other over the interval, 0 to T.
- Fig.(a)Given the set of coefficients {Sij} for j=1,2,3,....N operating as input generates the signal Si(t) for i=1,2,3,....M
- It consists of bank of N multipliers with each multiplier supplied with its own basis function followed by a summer.
- In the second stage i.e. fig.(b)Given the set of signals Si(t) for i=1,2,3,....M operating as input gives the set of coefficient {Sij} for j=1,2,3,....N
- The second scheme consists of a bank of N product integrators or correlators with a common input and with each one supplied with its own basis function.

Q2:*Explain step by step Gram Schmidt orthogonalization procedure.*

Answer:

Gram Schmidt orthogonalization step by step procedure:

- Step I: let $S_1(t),S_2(t),.....S_M(t)$ denote linearly independent signal, if not their exist a set of coefficients $a_1,a_2,......a_M$ not all equal to zero. i.e.
  $a_1S_1(t)+a_2S_2(t)+a_3S_3(t)+.....................+a_MS_M(t)=0$; $0 \leq t \leq T$

$$S_M(t) = -\left[\frac{a_1}{a_M}S_1(t) + \frac{a_2}{a_M}S_2(t) + \frac{a_3}{a_M}S_3(t) + ...+\frac{a_{M-1}}{a_M}S_M(t)\right]$$ for $a_M \neq 0$

- Step II:Construct a set of N orthonormal basis functions $\Phi_1(t), \Phi_2(t).........\Phi_N(t)$ from linearly independent signals $S_1(t), S_2(t), ...S_M(t)$,

  Its basis function, $\Phi_1(t) = \dfrac{S_1(t)}{\sqrt{E_1}}..................eq.1$

  Where E1=Energy of signal S1(t)

  $S_1(t) = \sqrt{E_1}\Phi_1(t)$

  $S_{11} = \sqrt{E_1}$

  $S_1(t) = S_{11}\Phi_1(t)$

  $\Phi_1(t)$ has unit energy if required. Next signal is $S_2(t)$,Coefficient S_{21} is given as

  $$S_{ij} = \int_0^T S_i(t)\Phi_j(t)dt$$

  $$S_{21} = \int_0^T S_2(t)\Phi_1(t)dt$$

  (i=2, j=1)

  Define a new intermediate functions

  $g_2(t)=S_2(t)-S_{21}\Phi_1(t)$ which is orthogonal to $\Phi_1(t)$,for $0 \le t \le T$

  Hence second basis function is given as $\Phi_2(t) = \dfrac{g_2(t)}{\sqrt{\int_0^T g_2^{\,2}(t)dt}}$

  $$\Phi_2(t) = \frac{S_2(t) - S_{21}\Phi_1(t)}{\sqrt{\int_0^T [S_2(t) - S_{21}\Phi_1(t)]^2 dt}}$$

  Consider denominator $\int_0^T [S_2(t) - S_{21}\Phi_1(t)]^2 dt$

$$\int_0^T [S_2(t) - S_{21}\Phi_1(t)]^2 dt = \int_0^T \{S_2^{\,2}(t) - 2S_2(t)S_{21}\Phi_1(t) + [S_{21}\Phi_1(t)]^2\} dt$$

$$= \int_0^T \{S_2^{\,2}(t) + S_{21}^{\,2}\Phi_1^{\,2}(t) - 2S_2(t)S_{21}\Phi_1(t)\} dt$$

$$= \int_0^T S_2^{\,2}(t)dt + \int_0^T S_{21}^{\,2}\Phi_1^{\,2}(t)dt - 2\int_0^T S_2(t)S_{21}\Phi_1(t)dt$$

$$S_{21} = \int_0^T S_2(t)\Phi_1(t)dt$$

$\Phi_1^{\,2}(t)dt = \Phi_1(t)\Phi_1(t)dt$, i=j=1

$$\int_0^T [S_2(t) - S_{21}\Phi_1(t)]^2 dt = E_2 + (S_{21}^{\,2} \times 1) - [2 \times S_{21}^{\,2}]$$

$$= E_2 + S_{21}^{\,2} - 2S_{21}^{\,2} = E_2 - S_{21}^{\,2}$$

$$\Phi_2(t) = \frac{g_2(t)}{\sqrt{\int_0^T g_2^{\,2}(t)dt}} = \frac{S_2(t) - S_{21}\Phi_1(t)}{\sqrt{\int_0^T [S_2(t) - S_{21}\Phi_1(t)]^2 dt}}$$

$$\Phi_2(t) = \frac{S_2(t) - S_{21}\Phi_1(t)}{\sqrt{E_2 - S_{21}^{\,2}}}$$

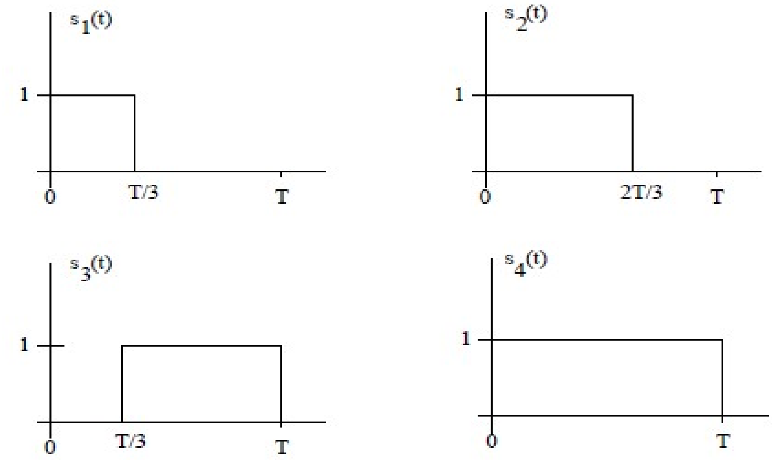
Similarly $g_i(t) = S_i(t) - \sum_{j=1}^{i-1} S_{ij}\Phi_j(t)$

Where Sij,for j=1,2,..........i-1

$$S_{ij} = \int_0^T S_i(t)\Phi_j(t)dt$$

Basis function is given by $\Phi_i(t) = \dfrac{g_i(t)}{\sqrt{\int_0^T g_i^2(t)dt}}$ i=1,2,3,...........N

Q3: *Figure displays the waveforms of four signals S1(t),S2(t),S3(t),S4(t) .*
(i)Use the Gram-Schmidt orthogonalization procedure; find an orthonormal basis for this set of signals
(ii) Construct the corresponding signal-space diagram.



Answer:
Step 1: This signal set is not linearly independent because S4(t)=S1(t)+S3(t)
S4(t) is dependent. Therefore S4(t) is linear combination of S1(t) and S3(t)
Hence we will use S1(t) ,S2(t),S3(t) toobtain the complete orthonormal set of basis functions.

M=4 no. of given signals

N=3 ,no. of orthonormal basis functions

N<M

We have to calculate $\Phi_1(t), \Phi_2(t), \Phi_3(t)$

Step 2: $\Phi_1(t) = \dfrac{S_1(t)}{\sqrt{E_1}}$

$$E_1 = \int_0^T S_1^{\,2}(t)dt$$

$$E_1 = \int_0^{T/3} S_1^{\,2}(t)dt + \int_{T/3}^{T} S_1^{\,2}(t)dt$$

$$\int_{T/3}^{T} S_1^{\,2}(t)dt = 0$$

$$E_1 = \int_0^{T/3} S_1^{\,2}(t)dt = \int_0^{T/3} (1)^2\,dt = \frac{T}{3}$$

$$\Phi_1(t) = \frac{S_1(t)}{\sqrt{E_1}}$$

$$\Phi_1(t) = \frac{1}{\sqrt{\frac{T}{3}}} = \sqrt{\frac{3}{T}}$$

$\Phi_1(t) = \sqrt{\dfrac{3}{T}}$ $\qquad\qquad 0 \le t \le T/3$

=0 $\qquad\qquad$ otherwise;

$$\Phi_2(t) = \frac{\mathrm{S}_2(\mathrm{t}) - \mathrm{S}_{21}\Phi_1(t)}{\sqrt{E_2 - S_{21}^{\;2}}}$$

$$S_{21} = \int_0^T \mathrm{S}_2(\mathrm{t})\Phi_1(t)dt$$

$$S_{21} = \int_0^{T/3} \mathrm{S}_2(\mathrm{t})\Phi_1(t)dt + \int_{T/3}^{2T/3} \mathrm{S}_2(\mathrm{t})\Phi_1(t)dt + \int_{2T/3}^{T} \mathrm{S}_2(\mathrm{t})\Phi_1(t)dt$$

$$S_{21} = \int_0^{T/3} (1)\sqrt{\frac{3}{T}}dt + \int_{T/3}^{2T/3} (1)\times 0 + 0$$

$$S_{21} = \int_0^{T/3} \sqrt{\frac{3}{T}}dt = \sqrt{\frac{3}{T}}\int_0^{T/3} (1)dt = \sqrt{\frac{T}{3}}$$

$$E_2 = \int_0^T \mathrm{S}_2^{\,2}(\mathrm{t})dt$$

$$E_2 = \int_0^{2T/3} \mathrm{S}_2^{\,2}(\mathrm{t})dt = \int_0^{2T/3} (1)dt = \frac{2T}{3}$$

$$\Phi_2(t) = \frac{\mathrm{S}_2(\mathrm{t}) - \mathrm{S}_{21}\Phi_1(t)}{\sqrt{E_2 - S_{21}^{\,2}}}$$

S_2(t)=from 0 to 2T/3

=0 to T/3 , T/3 to 2T/3

= 1 , 1

S_{21}=0 to T/3

$$= \sqrt{\frac{T}{3}}$$

$\Phi_1(t) = \sqrt{\frac{3}{T}}$ from 0 to T/3

$$\Phi_2(t) = \frac{1 - \sqrt{\frac{T}{3}}\sqrt{\frac{3}{T}}}{\sqrt{\frac{2T}{3} - \left(\sqrt{\frac{T}{3}}\right)^2}}$$

$$\Phi_2(t) = \frac{(1-1)+(1-0)}{\sqrt{\frac{2T}{3}-\left(\frac{T}{3}\right)}} = \frac{1}{\sqrt{\frac{T}{3}}} = \sqrt{\frac{3}{T}}$$

$\Phi_2(t) = \sqrt{\frac{3}{T}}$ for T/3 to 2T/3

=0 otherwise

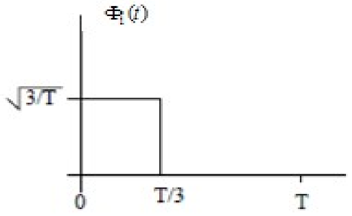
$$\Phi_3(t) = \frac{g_3(t)}{\sqrt{\int_0^T g_3{}^2(t)dt}}$$

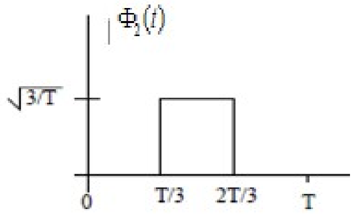
g3(t) for i=3

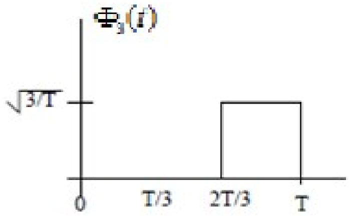
$$g_i(t) = \mathrm{S}_i\left(\mathrm{t}\right) - \sum_{j=1}^{i-1} S_{ij}\Phi_j(t)$$

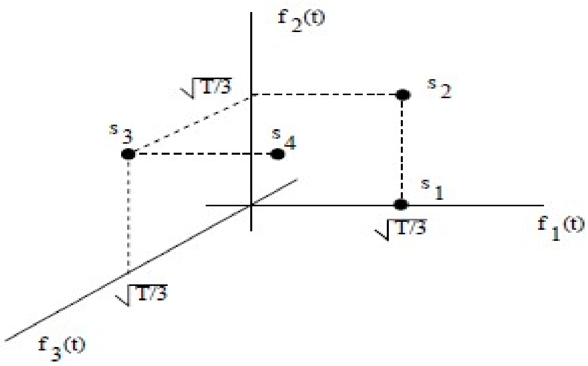
$$g_3(t) = \mathrm{S}_3\left(\mathrm{t}\right) - \sum_{j=1}^{2} S_{ij}\Phi_j(t)$$

$$g_3(t) = \mathrm{S}_3\left(\mathrm{t}\right) - S_{31}\Phi_1(t) - S_{32}\Phi_2(t)$$









$$s_1 = (\sqrt{T/3}, 0, 0)$$
$$s_2 = (\sqrt{T/3}, \sqrt{T/3}, 0)$$
$$s_3 = (0, \sqrt{T/3}, \sqrt{T/3})$$
$$s_4 = (\sqrt{T/3}, \sqrt{T/3}, \sqrt{T/3})$$

Q4: Prove that the impulse response of the optimum filter is time reversed and delayed version of the input signal Φ(t)

Answer:

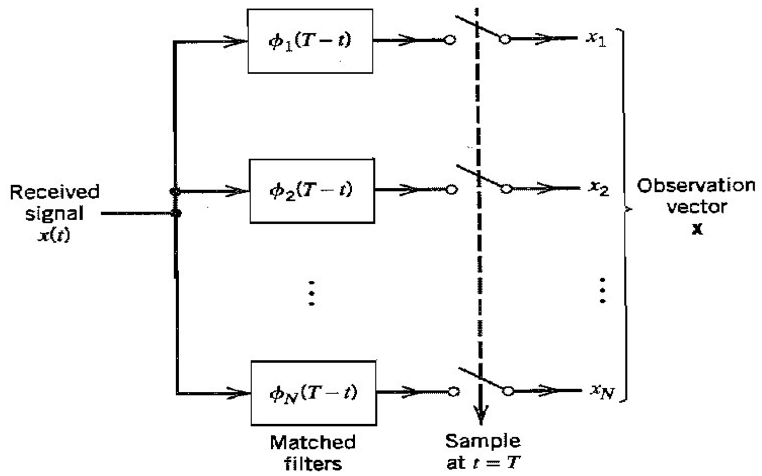
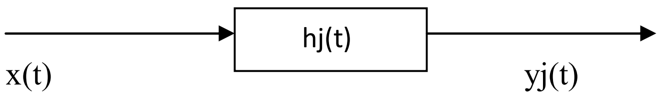


Fig.5.10:Detector part of matched Filter receiver

$\Phi_1(t), \Phi_2(t) .... \Phi_N(t)$ for 0≤t≤T

$\Phi_1(t), \Phi_2(t) .... \Phi_N(t)$ =0 for outside the interval



hj(t) =impulse response of filter

yj(t)=filter output

x(t)=Received signal at filter input

$$yj(t) = \int_{-\infty}^{\infty} x(\tau) hj(t-\tau) d\tau$$

Convolutional integral of x(t),hj(t)

$hj(t) = \Phi j(T-t)$

$hj(t-\tau) = \Phi j(T-(t-\tau))$

Replace $t = t-\tau$

$$hj(t-\tau)=\Phi j(T-t+\tau)$$

$$yj(t)=\int_{-\infty}^{\infty} x(\tau)\Phi j(T-t+\tau)d\tau$$

Sample at time t=T
Replace t=T

$$yj(T)=\int_{-\infty}^{\infty} x(\tau)\Phi j(T-T+\tau)d\tau$$

$$yj(T)=\int_{-\infty}^{\infty} x(\tau)\Phi j(\tau)d\tau$$

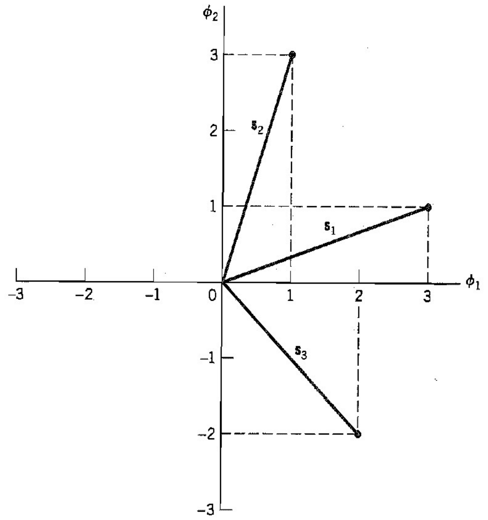
$\Phi j(t)=0$ outside $0 \le t \le T$

$$yj(T)=\int_{-0}^{T} x(\tau)\Phi j(\tau)d\tau$$

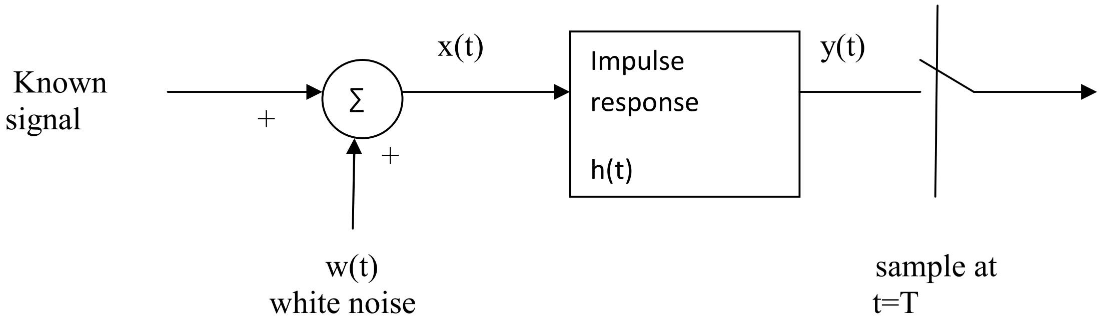
$$y_j(T)=x_j$$

xj=jth correlater output produced by x(t)
A filter whose impulse response is time reversed and delayed version of signal $\Phi j(t)$ is called as matched filter receiver.

Q5:*Illustrate the geometric representation of signals for the case when N=2 and M=3*
Answer:



Q6: *Derive Signal to noise ratio of matched filter receiver.*
Answer:



w(t)=additive white Gaussian noise zero mean
PSD=Power Spectral Density=No/2

$x(t) = \Phi(t) + w(t)$

$y(t) = \Phi o(t) + n(t)$

$\Phi o(t), n(t)$ are the signal and noise component of x(t)

$$(SNR)o = \frac{|\Phi o(t)|^2}{E[n^2(t)]}$$

E=Expected mean of noise

- Maximization occurs when the filter is matched to the known signal at the input
- $\Phi(f)$ =fourier transform of $\Phi(t)$
- H(f)=transfer function of the filter.
- $\Phi o(t) = H(f).\Phi(f)$
- Taking inverse fourier transform T

  $$\Phi o(t) = \int_{-\infty}^{\infty} H(f).\Phi(f).\exp(j2\Pi fT)df$$

  Sample at t=T

  $$|\Phi o(T)|^2 = |\int_{-\infty}^{\infty} H(f).\Phi(f).\exp(j2\Pi fT)df|^2$$

  Consider effect of noise alone at output,w(t)

  S_N(f)=PSD of output noise ,n(t)

  N_0/2= PSD of input noise,w(t)

$$S_N(f) = \frac{N_0}{2}|H(f)|^2$$

Average power and output noise ,n(t)

$$E[n^2(t)] = \int_{-\infty}^{\infty} S_N(f)df$$

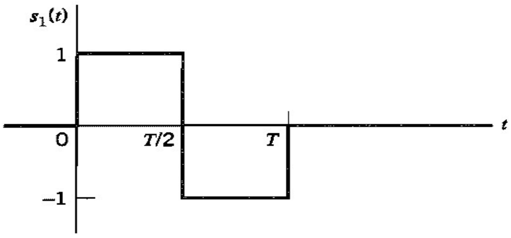
$$E[n^2(t)] = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 df$$

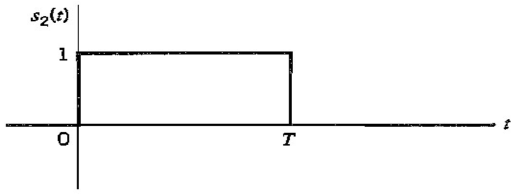
$$(SNR)o = \frac{|\int_{-\infty}^{\infty} H(f).\Phi(f).\exp(j2\Pi fT)df|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty}|H(f)|^2 df}$$

Q7: *Figure displays the waveforms of four signals S1(t),S2(t)*

(i)Use the Gram-Schmidt orthogonalization procedure, find an orthonormal basis for this set of signals

(ii) Construct the corresponding signal-space diagram.





Answer:

$$\Phi_1(t) = \frac{S_1(t)}{\sqrt{E_1}}$$

$$E_1 = \int_0^T S_1^{\,2}(t)dt$$

$$E_1 = \int_0^{T/2} (1)^2 dt + \int_{T/2}^{T} (-1)^2 dt$$

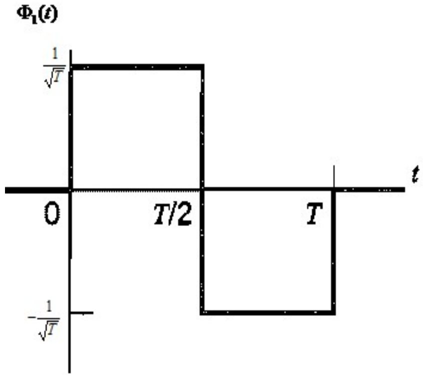
$$E_1 = 1\left(\frac{T}{2} - 0\right) + 1\left(T - \frac{T}{2}\right)$$

$$E_1 = 1\left(\frac{T}{2}\right) + 1\left(\frac{T}{2}\right) = T$$

$$\Phi_1(t) = \frac{S_1(t)}{\sqrt{E_1}}$$

$$\Phi_1(t) = \frac{1}{\sqrt{T}} \quad \text{for} \quad 0 \le t \le T/2$$

$$\Phi_1(t) = -\frac{1}{\sqrt{T}} \quad \text{for} \quad T/2 \le t \le T$$



$$\Phi_2(t) = \frac{S_2(t) - S_{21}\Phi_1(t)}{\sqrt{E_2 - S_{21}{}^2}} \backslash$$

$$S_{21} = \int_0^T S_2(t)\Phi_1(t)\,dt$$

UNIT No IV

1 DIGITAL MODULATION TECHNIQUES

Q1. *Explain coherent Binary Phase Shift Keying(BPSK) modulation with the help of transmitter and receiver block diagram*

Answer:

BPSK Transmitter:

The block diagram of coherent Binary Phase Shift Keying Transmitter consists of Polar non-return to zero level encoder and the Product modulator which has the carrier sine wave as one input and the binary sequence as the other input as shown in figure 2.1. The carrier and the timing pulses used to generate the desired Binary PSK signal S(t) .

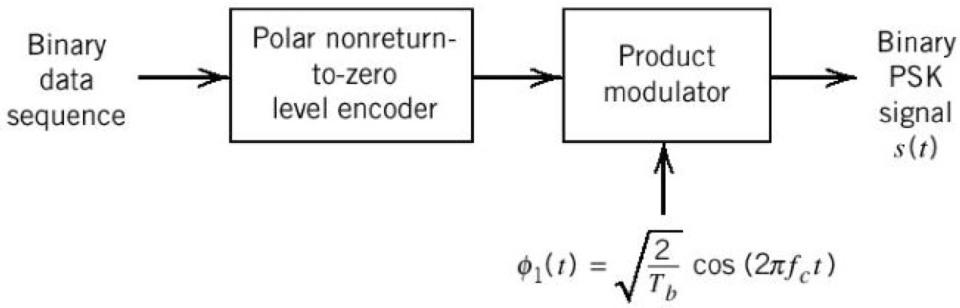


Figure 2.1: Coherent BPSK Transmitter

The modulation of BPSK is done using a Product Modulator, which multiplies the two signals applied at the input, for a ‘0’ binary input, the phase will be 0° and for a ‘1’, the phase reversal is of 180°.Following figure 2.2 shows the representation of BPSK Modulated output wave along with its given input.

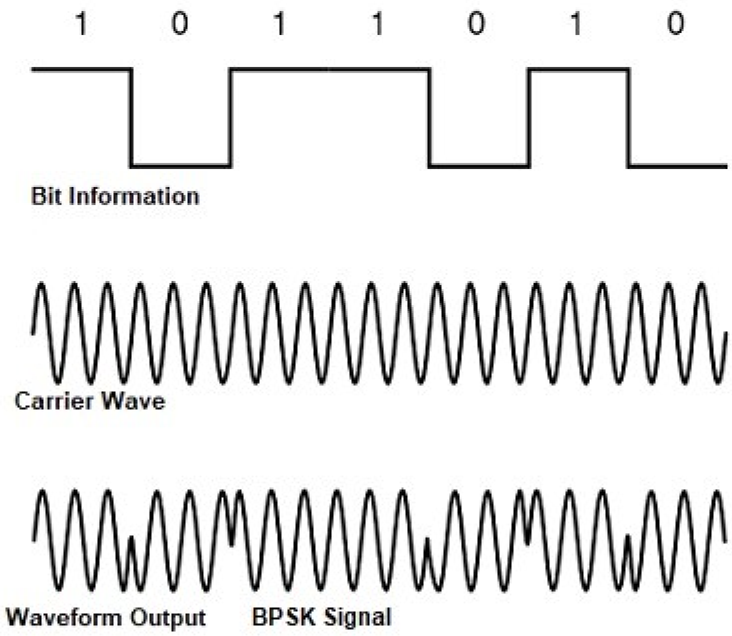


Figure 2.2: BPSK Modulated output waveform

BPSK Receiver

To detect the original binary sequence, we apply noisy BPSK signal to correlator as shown in the block diagram of BPSK Receiver. The correlator is applied with locally generated signal

$\varphi_1(t)$. The correlator output x_1, is compared with the threshold zero volts. If x1 > 0 the receiver decides in favor of 1 otherwise if x1 < 0 the receiver decides in favor of 0.

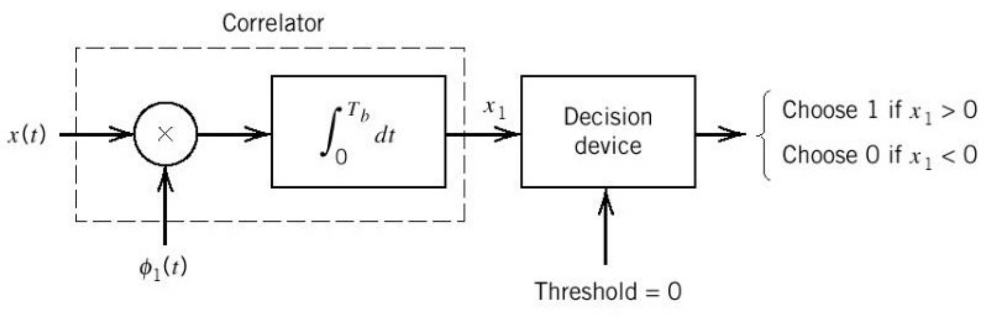


Figure 2.3: Coherent BPSK Receiver

Q2. *Derive co-ordinates and draw signal space diagram for Binary Phase Shift Keying(BPSK) system*

In the coherent BPSK system ,the pair of signals $s_1(t)$ and $s_2(t)$ used to define binary symbols 1 and 0 respectively

$$S_1(t) = \sqrt{\frac{2E_b}{T_b}} * cos2\pi f_c t \qquad \textbf{(1)}$$

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} * [cos2\pi f_c t + \pi] \ = - \sqrt{\frac{2E_b}{T_b}} * cos2\pi f_c t \qquad (2)$$

A pair of sinusoidal signals defined by equation (1) and (2) are differ only in phase-shift of 180 degrees also called as antipodal signals. From the above pair of equations it is clear that the BPSK system has only one basis function, □1 (t) is defined as:

$$\varphi_1(t) = \sqrt{\frac{2}{T_b}} * cos2\pi f_c t \quad for\, 0 \le t \le T_b \qquad (3)$$

Then we may express the transmission signals $s_1(t)$ and $s_2(t)$ in terms of $\varphi_1(t)$ as:

$$s_1(t) = \sqrt{E_b} * \varphi(t) \qquad (4)$$

$$and \quad s_2(t) = -\sqrt{E_b} * \varphi(t) \qquad (5)$$

The coherent BPSK system is characterized using signal space that is one dimensional(i.e. N=1) and with a signal constellation consisting of two message points (i.e. M=2) .

The co-ordinates of message signals are defined as:

$$s_{11} = \int_0^{T_b} s_1(t) * \varphi_1(t) dt = +\sqrt{E_b} \qquad (6)$$

$$and \quad s_{21} = \int_0^{T_b} s_2(t) * \varphi_1(t) dt = -\sqrt{E_b} \qquad (7)$$

Message point corresponding to signal $s_1(t)$ is $s_{11} = +\sqrt{Eb}$ and Message point corresponding to signal $s_1(t)$ is $s_{21} = -\sqrt{Eb}$.

The Signal Space Diagram for BPSK is shown in figure 2.4 given below.

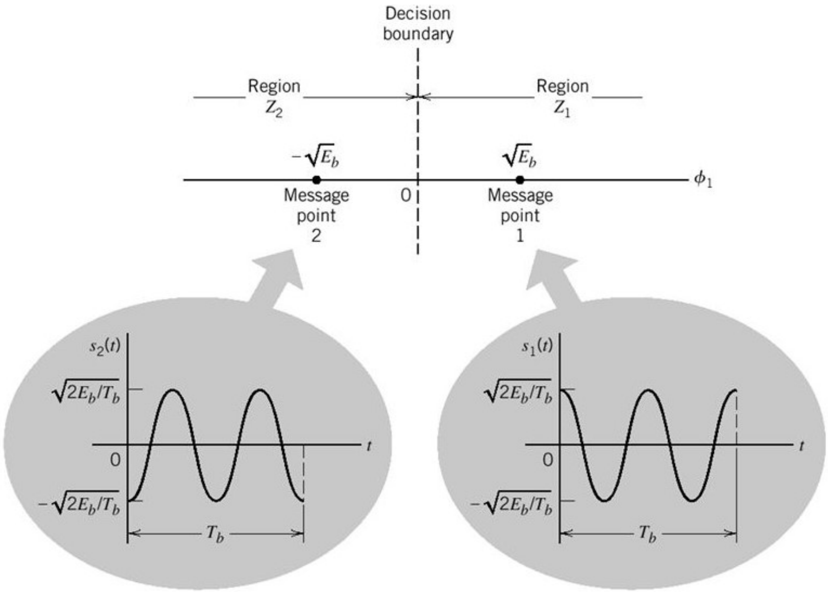


Figure 2.4: Signal Space Diagram for BPSK

Q 3. *Explain coherent* Quadrature *Phase Shift Keying(QPSK) modulation with the help of transmitter and receiver block diagram*

Answer:

Quadrature Phase Shift Keying (QPSK) Transmitter :

The block diagram of shows that the incoming binary data sequence is first converted into a polar form by non-return to zero level encoder. Thus symbol 1 and 0 are represented by + $\sqrt{E_b}$ and - $\sqrt{E_b}$ respectively. The binary wave is then divided by de-multiplexer into two binary waves consisting of even and odd numbered input bits.These two binary waves are denoted by $a_1(t)$ and $a_2(t)$.These two binary waves are used to modulate the pair of orthonormal basis function $\varphi_{1(t)}$ and $\varphi_{2(t)}$ respectively. The result is a pair of binary- PSK signals. Finally two binary PSK signals are added to form a QPSK signal.

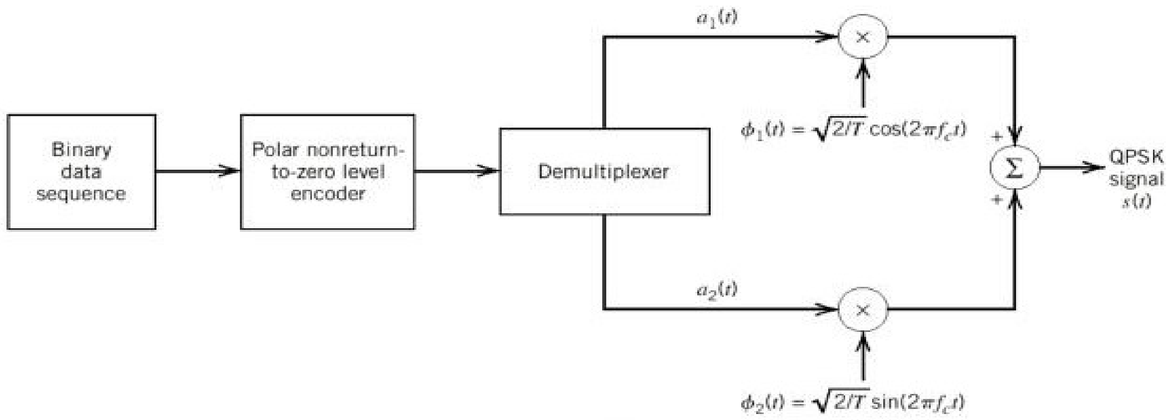


Figure 2.5: Coherent QPSK Transmitter

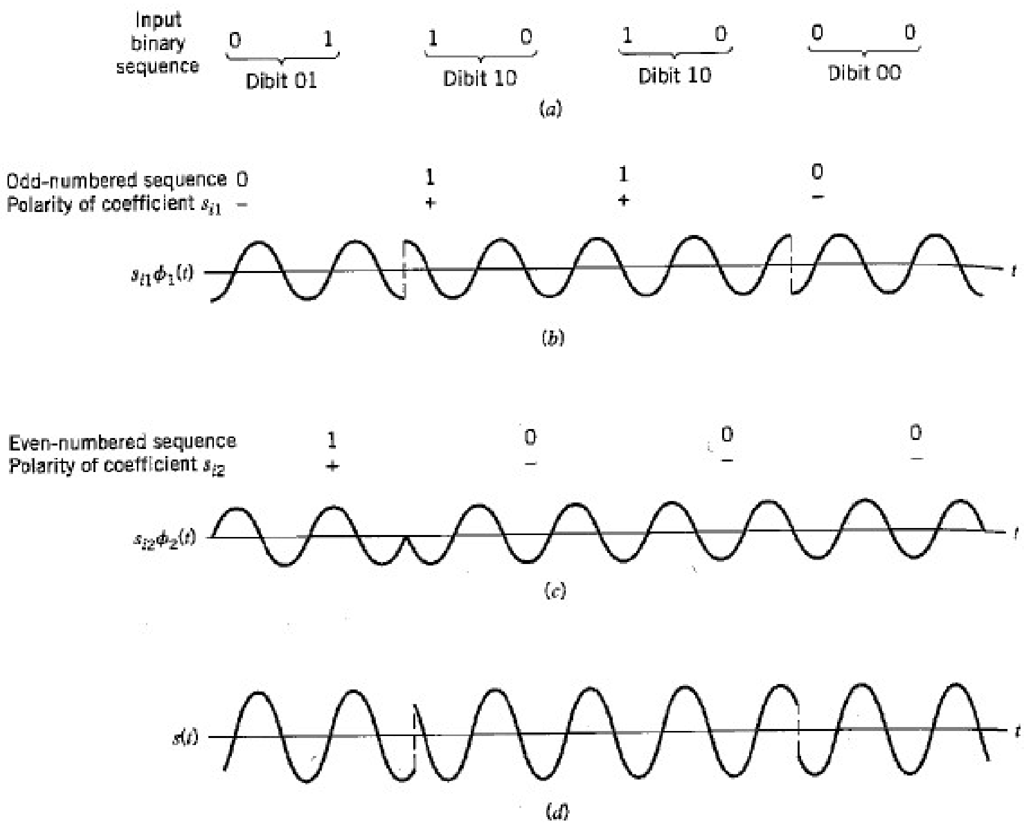


Figure 2.6: a) Input binary sequence b)Odd numbered bits of input sequence c) EVEN numbered bits of input sequence d)QPSK waveform

Quadrature Phase Shift Keying (QPSK) Receiver:

The QPSK receiver consists of a pair of correlators with common input received signals and supplied with a locally generated coherent reference signals $\varphi_{1(t)}$ and $\varphi_{2(t)}$ respectively. The correlators outputs x_1 and x_2 are compared with a threshold of zero. If $x_1>0$,a decision is made in favor of symbol 1 for in phase channel output, but If $x_1 < 0$,a decision is made in favor of symbol 0 .Similarly If $x_2>0$,a decision is made in favor of symbol 1 for in quadrature channel output, but If $x_2 < 0$,a decision is made in favor of symbol 0.Finally the two binary sequences at the in phase and quadrature channel outputs are combined in a multiplexer to reproduce the original binary sequence at the transmitter input.

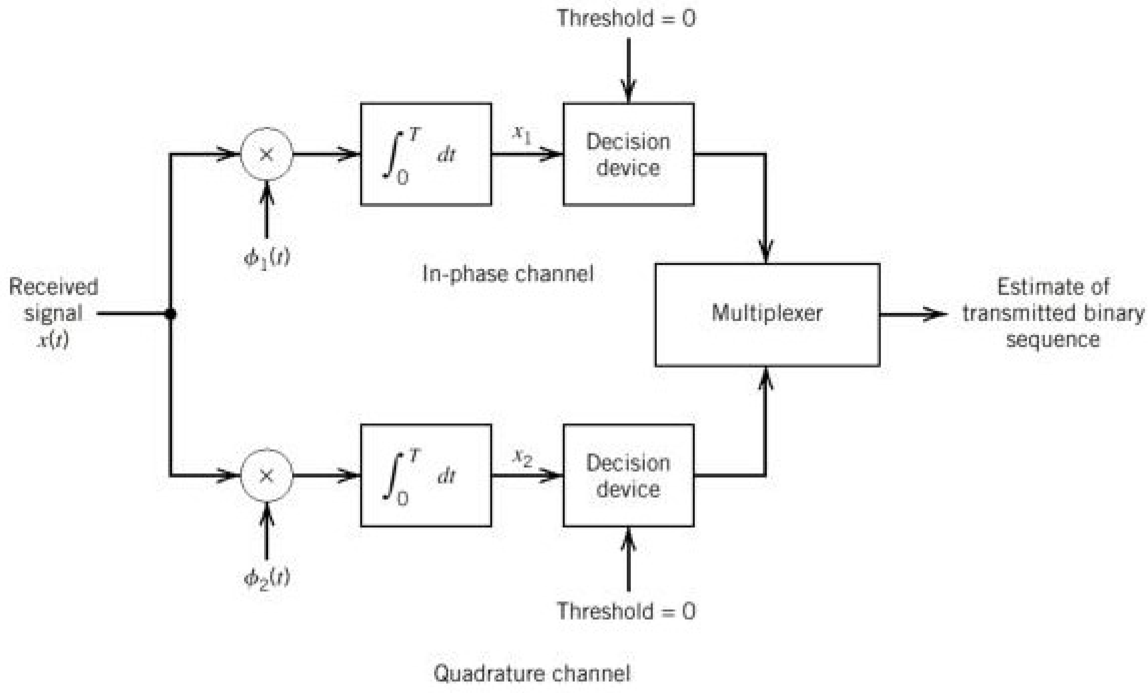


Figure 2.7: Coherent QPSK Receiver

Q4. Derive co-ordinates and draw signal space diagram for Quadrature Phase Shift Keying(QPSK) system.

Answer:

In QPSK,the phase of carrier takes one of the four equally spaced values $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$ and $\frac{7\pi}{4}$. For this set of values we may define the transmitted signal as:

$$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} * [\cos 2\pi f_c t + (2i-1)\frac{\pi}{4}], & 0 \le t \le T \\ 0 & elsewhere \end{cases} \quad (1)$$

Where

i=1,2,3,4

E-Transmitted Signal Energy per symbol

T-Symbol Duration

fc = nc/T where nc is some fixed integer

Now each possible value of phase corresponding to dibits (gray encoded set of dibits) :10,00,01 and 11 as shown in a table below.

Gray coded input dibits	Phase of QPSK signals(radians)	Co ordinates of message points	
		S_{i1}	S_{i2}
10	Π/4	$+\sqrt{E}$	$-\sqrt{E}$

00	3Π/4	$-\sqrt{E}$	$-\sqrt{E}$
01	5Π/4	$-\sqrt{E}$	$+\sqrt{E}$
11	7Π/4	$+\sqrt{E}$	$+\sqrt{E}$

Signal space Diagram of QPSK:

$$S_i(t) = \left\{ \sqrt{\frac{2E}{T}} * \cos\left[(2i-1)\frac{\pi}{4}\right] * \cos 2\pi f_c t - \sqrt{\frac{2E}{T}} * \sin\left[(2i-1)\frac{\pi}{4} * \sin 2\pi f_c t \right.\right. \qquad (2)$$

Where

i=1,2,3,4

There are two orthonormal basis functions $\varphi_{1(t)}$ and $\varphi_{2(t)}$ contained in the above expression of $S_i(t)$. These are defined as:

$$\varphi_1(t) = \sqrt{\frac{2E}{T}} * \cos 2\pi f_c t, \qquad 0 \le t \le T \qquad (3)$$

$$\varphi_2(t) = \sqrt{\frac{2E}{T}} * \sin 2\pi f_c t, \qquad 0 \le t \le T \qquad (3)$$

There are four message points and the associated signal vectors are defined as:

$$S_i = \begin{bmatrix} +\sqrt{E} * \cos\left((2i-1)\frac{\pi}{4}\right) \\ -\sqrt{E} * \sin\left((2i-1)\frac{\pi}{4}\right) \end{bmatrix}, \qquad i = 1,2,3,4 \qquad (4)$$

Accordingly a QPSK signal has two dimensional signal constellation(i.e. N=2) and four message points (i.e. M=4) as shown in figure 2.8 .

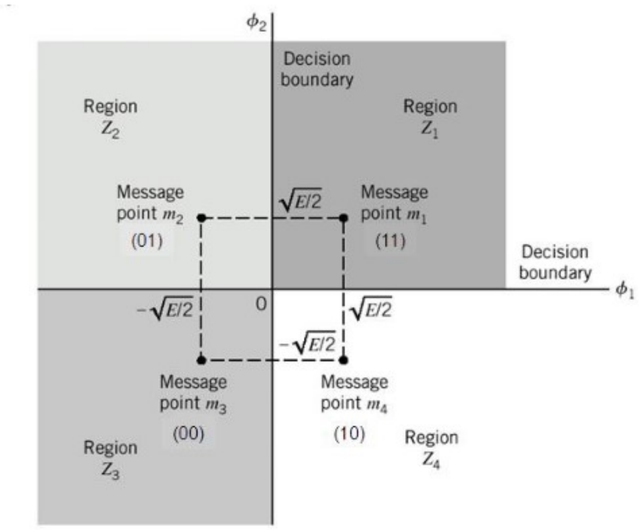


Figure2.8: Signal space Diagram of QPSK

Q5. *Explain coherent Binary Frequency Shift Keying(BFSK) modulation with the help of transmitter and receiver block diagram*

Answer:

Binary Frequency Shift Keying (BFSK) transmitter:

To generate BFSK signal a binary data sequence is first applied to on-off level encoder as shown in figure 2.9, at the output of which symbol 1 is represented by constant amplitude $\sqrt{E_b}$ volts and symbol 0 is by zero volts. When symbol 1 is at the input the oscillator with frequency f_1 at the upper channel is switched on while the oscillator with frequency f_2 at the lower channel is switched off ,with the result that the frequency f_1 is transmitted. Conversely , When symbol 0 is at the input the oscillator with frequency f_1 at the upper channel is switched off while the oscillator with frequency f_2 at the lower channel is switched on ,with the result that the frequency f_2 is transmitted. The two frequencies f_1 and f_2 are chosen as the integral multiple of the bit rate $1/T_b$.The two multiplier are supplied with orthonormal basis functions $\varphi_{1(t)}$ and $\varphi_{2(t)}$ respectively. Finally output of two multipliers are added to form Binary FSK signal.

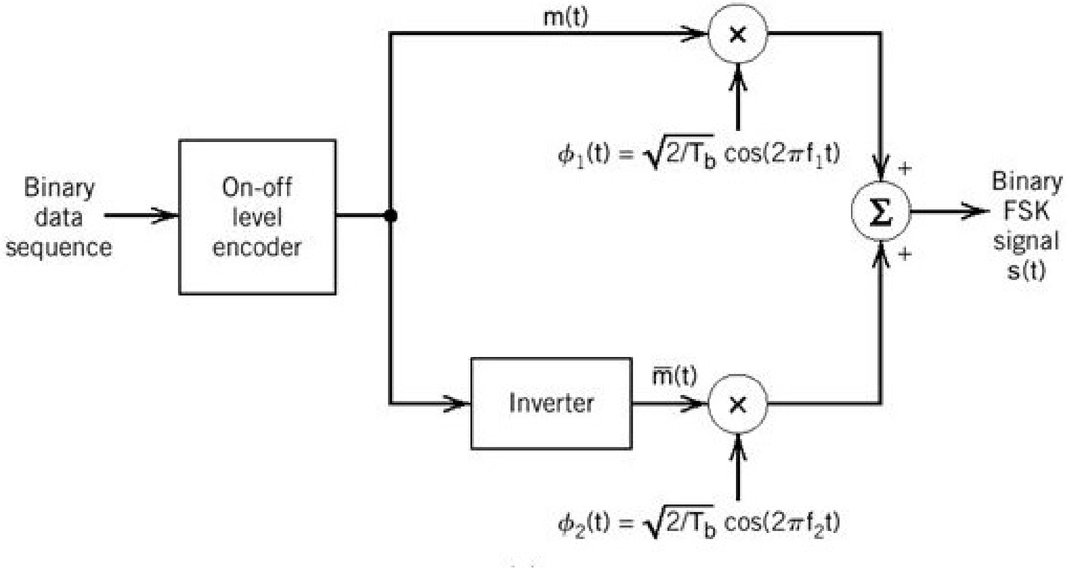


Figure 2.9: Coherent Binary Frequency Shift Keying (BFSK) transmitter

Binary Frequency Shift Keying (BFSK) receiver:

To detect the original binary sequence BPSK noisy signal x(t) is applied to the a pair of correlators with locally generated signals $\varphi_{1(t)}$ and $\varphi_{2(t)}$ .The correlated outputs are subtracted, one from the other and the resulting difference y is compared with the threshold zero volts. If y>0 ,the receiver decides in favor of symbol 1 or If y<0 ,the receiver decides in favor of symbol 1.If y is exactly 0 the receiver makes random guess 1 or 0.

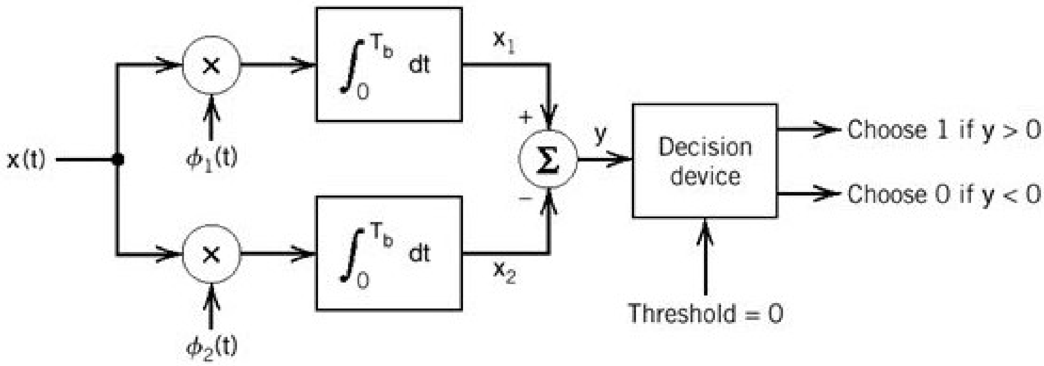


Figure 2.10: Coherent Binary Frequency Shift Keying (BFSK) Receiver

Q6. *Derive co-ordinates and draw signal space diagram for Binary Frequency Shift Keying (BFSK) system.*

Answer:

In Binary FSK system, symbols 1 and 0 are distinguished from each other by transmitting one of two sinusoidal waves that differ in frequency by fixed amount. A typical pair of sinusoidal waves is described by

$$S_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} * \cos(2\pi f_i t) & 0 \le t \le T \\ 0 & elsewhere \end{cases} \qquad (1)$$

Where i=1,2 and E_b energy transmitted per bit, the transmitted frequency is

$$f_i = \frac{n_c + i}{T_b} \qquad \textit{for some fixed integer nc and } i = 1{,}2 \qquad (2)$$

Thus symbol 1 is represented by $S_1(t)$ and symbol 0 by $S_2(t)$.

From equation 1 and equation 2 are $S_1(t)$ and $S_2(t)$ are orthogonal and the set of orthonormal basis function is expressed as:

$$\varphi_i(t) = \begin{cases} \sqrt{\frac{2}{T_b}} * \cos(2\pi f_i t) & 0 \le t \le T \\ 0 & elsewhere \end{cases} \qquad (3)$$

Where i=1,2

Correspondingly, the coefficient S_{ij} for i=1,2 1nd j=1,2 is defined as:

$$S_{ij} = \int_0^{T_b} S_i(t) * \varphi_j(t)\,dt$$

$$= \int_0^{T_b} \left(\sqrt{\frac{2E_b}{T}} * \cos(2\pi f_i t) \right) * \sqrt{\frac{2}{T}} * \cos\,(2\pi f_i t)) \qquad (4)$$

$$= \begin{cases} \sqrt{E_b} & i = j \\ 0 & i \neq j \end{cases}$$

BPSK system is characterised by a single space i.e. two dimensional(i.e. N=2) and two message points(i.e. M=2) as shown in figure 2.11. The two message points are defined as:

$$S_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix} \qquad (5)$$

$$\mathbf{s}_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix} \qquad (6)$$

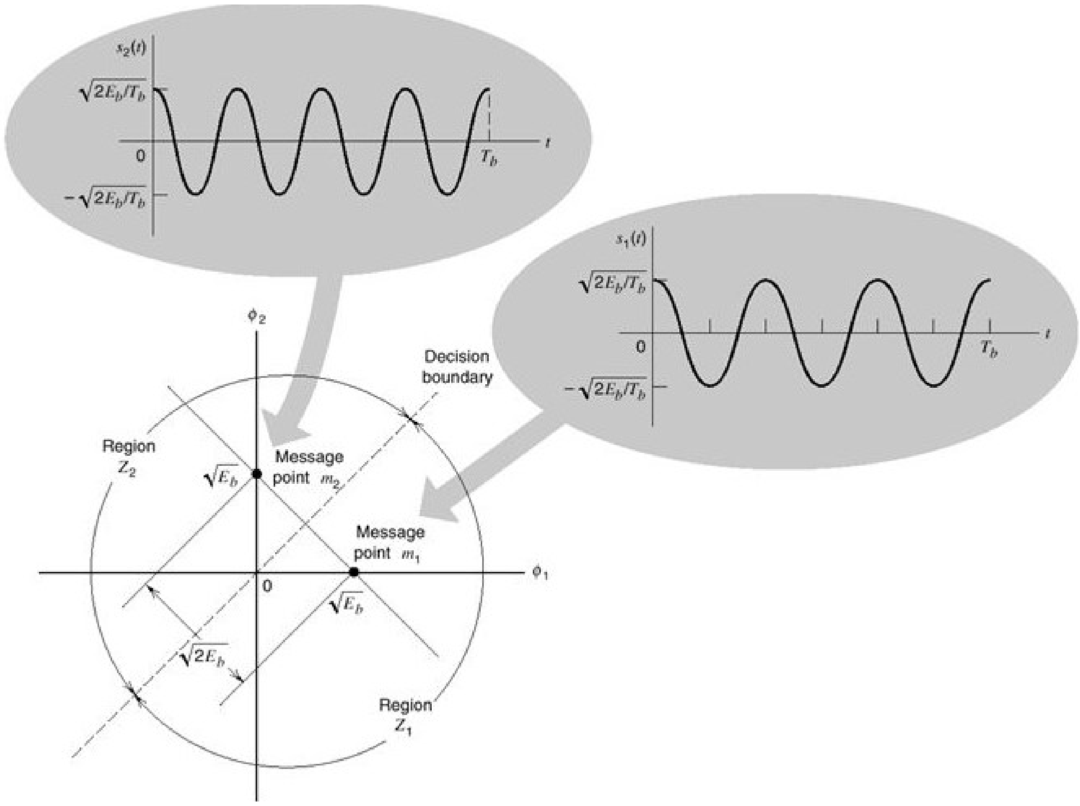


Figure 2.11: Signal Space Diagram of Binary Frequency Shift Keying (BFSK)

Q7. *Describe Frequency Division Multiplexing(FDM) in detail.*

Answer:

Frequency Division Multiplexing(FDM)

Whenever the bandwidth of a medium linking two devices is greater than the bandwidth needs of the devices, the link can be shared. Multiplexing is the set of techniques that allows the simultaneous transmission of multiple signals across a single data link.

In a multiplexed system, *n* lines share the bandwidth of one link. Figure 2.12 shows the basic format of a multiplexed system. The lines on the left direct their transmission streams to a multiplexer (MUX), which combines them into a single stream (many-to-one). At the receiving end, that stream is fed into a demultiplexer (DEMUX), which separates the stream back into its component transmissions (one-to-many) and directs them to their corresponding lines. In the figure, the word link refers to the physical path. The word channel refers to the portion of a link that carries a transmission between a given pair of lines. One link can have many *(n)* channels.

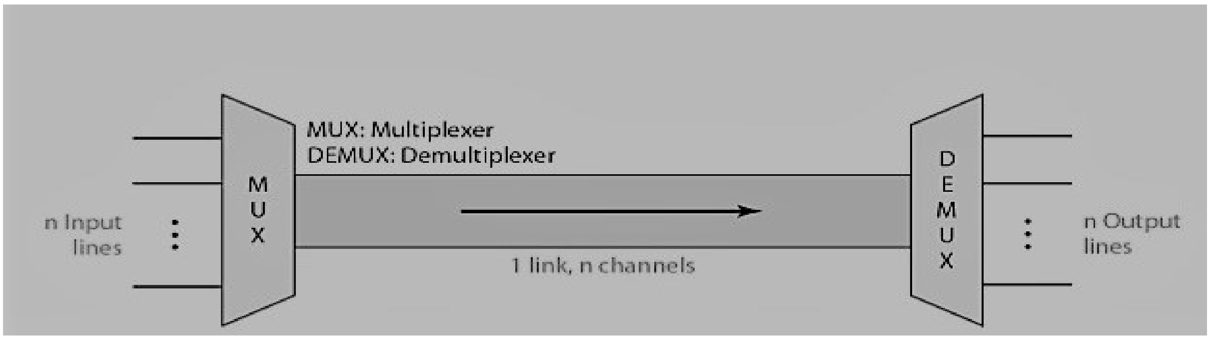


Figure 2.12 : Dividing a link into channels

There are three basic multiplexing techniques: frequency-division multiplexing, wavelength- division multiplexing, and time-division multiplexing. The first two are techniques designed for analog signals, the third, for digital signals.

Frequency-division multiplexing (FDM) is an analog technique that can be applied when the bandwidth of a link (in hertz) is greater than the combined bandwidths of the signals to be transmitted. In FDM, signals generated by each sending device modulate different carrier frequencies. These modulated signals are then combined into a single composite signal that can be transported by the link. Carrier frequencies are separated by sufficient bandwidth to accommodate the modulated signal. These bandwidth ranges are the channels through which the various signals travel. Channels can be separated by strips of unused bandwidth-guard bands-to prevent signals from overlapping. In addition, carrier frequencies must not interfere with the original data frequencies.

Multiplexing Process:
Figure 2.13 is a conceptual illustration of the multiplexing process. Each source generates a signal of a similar frequency range. Inside the multiplexer, these similar signals modulates different carrier frequencies. The resulting modulated signals are then combined into a single composite signal that is sent out over a media link that has enough bandwidth to accommodate it.

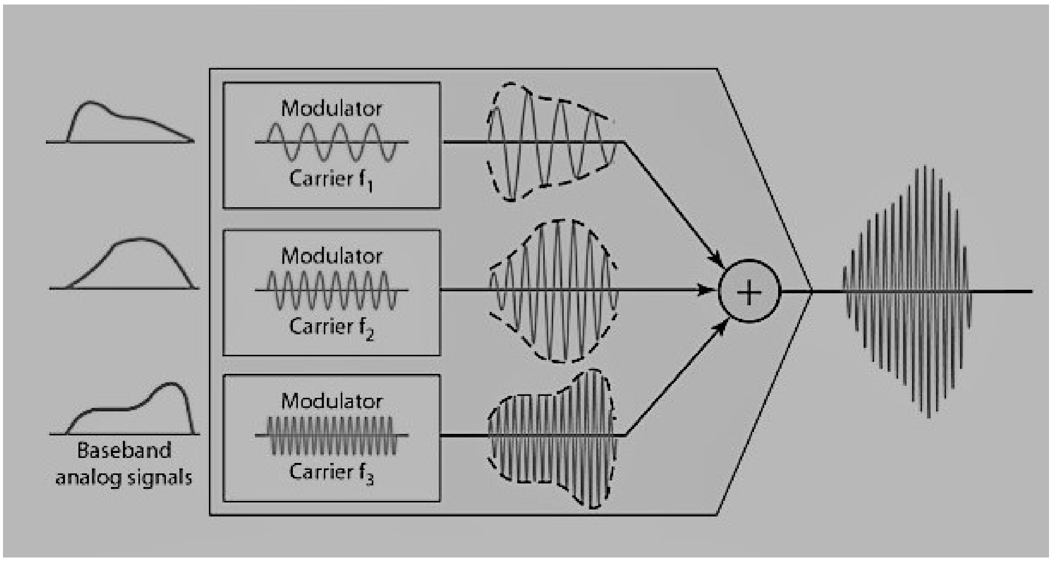


Figure 2.13: FDM Multiplexing Process

Demultiplexing Process:

The demultiplexer uses a series of filters to decompose the multiplexed signal into its constituent component signals. The individual signals are then passed to a demodulator that separates them from their carriers and passes them to the output lines. Figure 2.14 is a conceptual illustration of demultiplexing process.

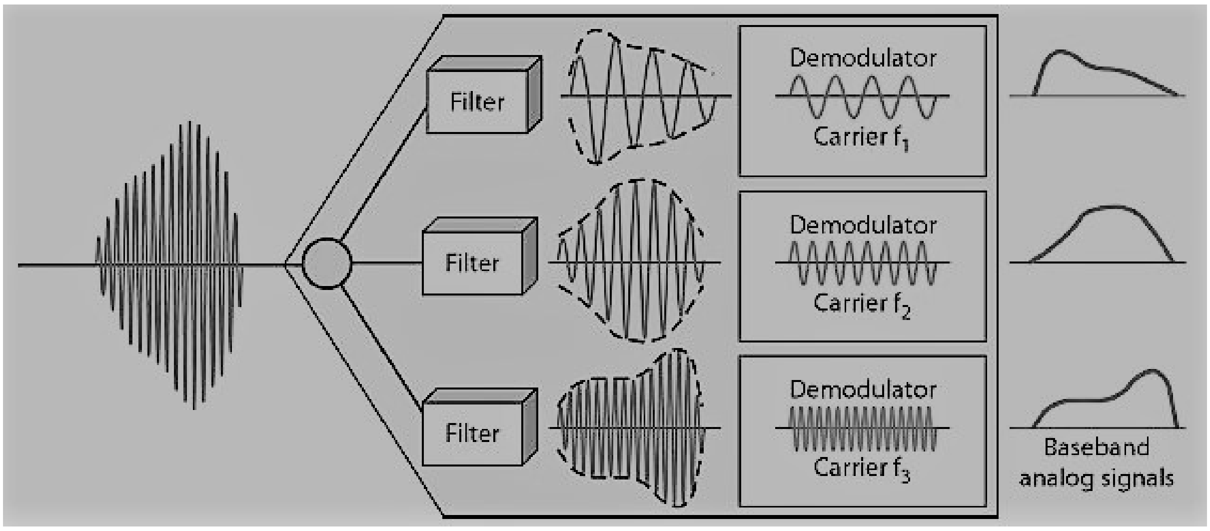


Figure 2.14: FDM Demultiplexing Process

Q8. *Describe Time Division Multiplexing(FDM) in detail.*

Answer:

Time Division Multiplexing(FDM):

Whenever the bandwidth of a medium linking two devices is greater than the bandwidth needs of the devices, the link can be shared. Multiplexing is the set of techniques that allows the simultaneous transmission of multiple signals across a single data link.

In a multiplexed system, *n* lines share the bandwidth of one link. Figure 2.15 shows the basic format of a multiplexed system. The lines on the left direct their transmission streams to a multiplexer (MUX), which combines them into a single stream (many-to-one). At the receiving end, that stream is fed into a demultiplexer (DEMUX), which separates the stream back into its component transmissions (one-to-many) and directs them to their corresponding lines. In the figure, the word link refers to the physical path. The word channel refers to the portion of a link that carries a transmission between a given pair of lines. One link can have many *(n)* channels.

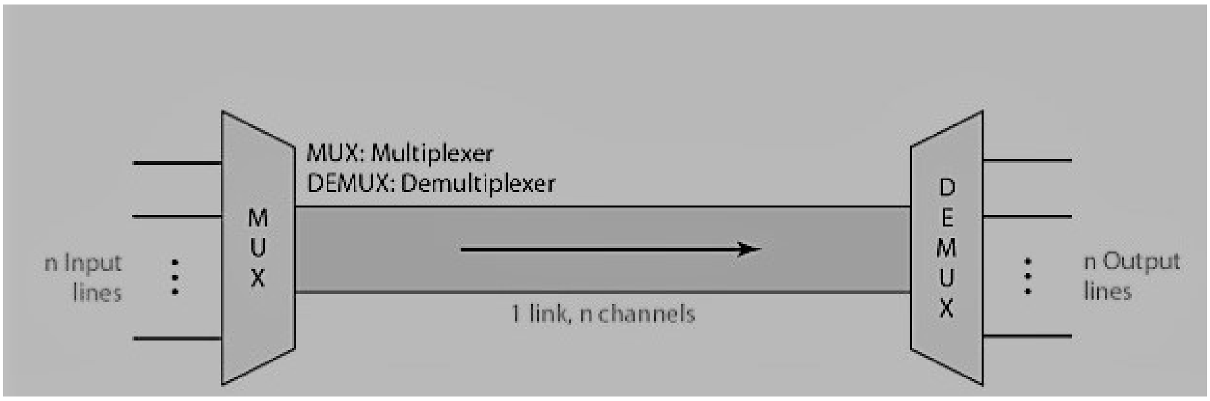


Figure 2.15 : Dividing a link into channels

There are three basic multiplexing techniques: frequency-division multiplexing, wavelength- division multiplexing, and time-division multiplexing. The first two are techniques designed for analog signals, the third, for digital signals.

Time-division multiplexing (TDM) is a digital process that allows several connections to share the high bandwidth of a linle Instead of sharing a portion of the bandwidth as in FDM, time is shared. Each connection occupies a portion of time in the link. Figure 2.16 gives a conceptual view of TDM. Note that the same link is used as in FDM; here, however, the link is shown sectioned by time rather than by frequency. In the figure, portions of signals 1,2,3, and 4 occupy the link sequentially.

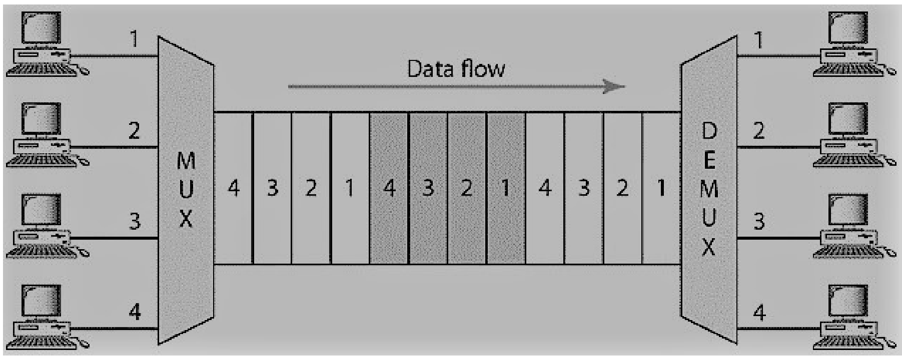


Figure 2.16: Time-Division Multiplexing

Note that in Figure 6.12 we are concerned with only multiplexing, not switching. This means that all the data in a message from source 1 always go to one specific destination, be it 1, 2, 3, or 4. The delivery is fixed and unvarying, unlike switching.

Q9. *Describe Wavelength Division Multiplexing(FDM) in detail.*

Answer:

Wavelength Division Multiplexing(FDM):

Whenever the bandwidth of a medium linking two devices is greater than the bandwidth needs of the devices, the link can be shared. Multiplexing is the set of techniques that allows the simultaneous transmission of multiple signals across a single data link.

In a multiplexed system, *n* lines share the bandwidth of one link. Figure 2.17 shows the basic format of a multiplexed system. The lines on the left direct their transmission streams to a multiplexer (MUX), which combines them into a single stream (many-to-one). At the receiving end, that stream is fed into a demultiplexer (DEMUX), which separates the stream back into its component transmissions (one-to-many) and directs them to their corresponding lines. In the figure, the word link refers to the physical path. The word channel refers to the portion of a link that carries a transmission between a given pair of lines. One link can have many *(n)* channels.

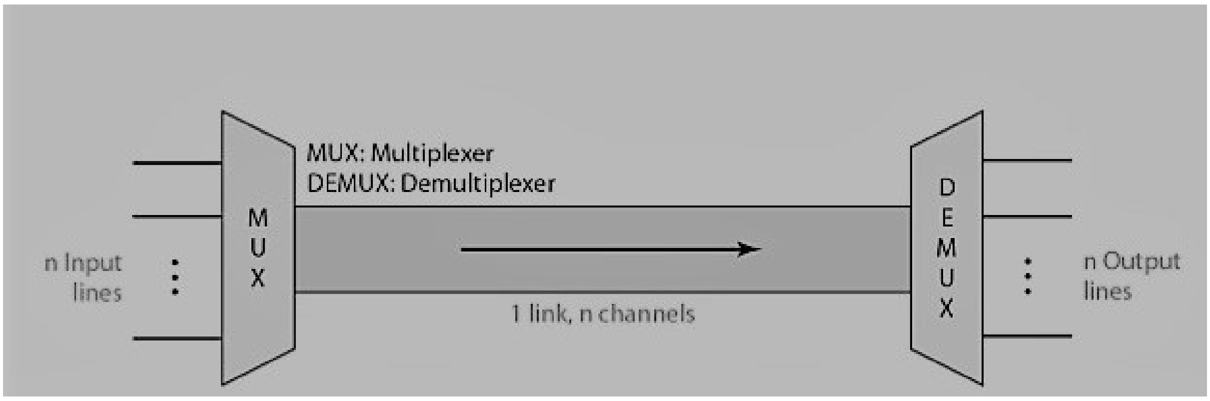


Figure 2.17 : Dividing a link into channels

There are three basic multiplexing techniques: frequency-division multiplexing, wavelength- division multiplexing, and time-division multiplexing. The first two are techniques designed for analog signals, the third, for digital signals.

Wavelength-division multiplexing (WDM) is designed to use the high-data-rate capability of fiber-optic cable. The optical fiber data rate is higher than the data rate of metallic transmission cable. Using a fiber-optic cable for one single line wastes the available bandwidth. Multiplexing allows us to combine several lines into one. WDM is conceptually the same as FDM, except that the multiplexing and demultiplexing involve optical signals transmitted through fiber-optic channels. The idea is the same: We are combining different signals of different frequencies. The difference is that the frequencies are very high. Figure 2.18 gives a conceptual view of a WDM multiplexer and demultiplexer. Very narrow bands of light from different sources are combined to make a wider band of light. At the receiver, the signals are separated by the demultiplexer.

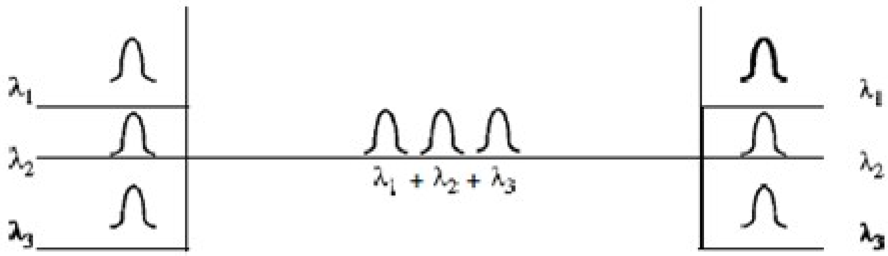


Figure 2.18 :WDM multiplexer

Although WDM technology is very complex, the basic idea is very simple. We want to combine multiple light sources into one single light at the multiplexer and do the reverse at the demultiplexer. The combining and splitting of light sources are easily handled by a prism.

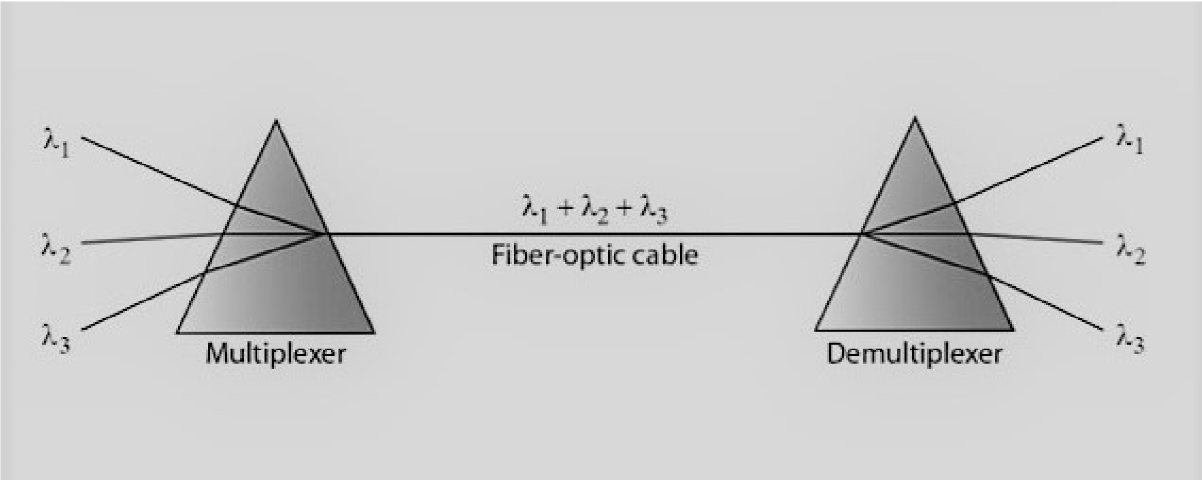


Figure2.19: Prisms in WDM Multiplexing and Demultiplexing

One application of WDM is the SONET network in which multiple optical fiber lines are multiplexed and demultiplexed.

Q10. *Describe Orthogonal Frequency Division multiplexing (OFDM).*

Answer:

Orthogonal Frequency Division multiplexing (OFDM):

OFDM modulation technique is generated through the use of complex signal processing approaches such as Fast Fourier Transforms (FFTs) and inverse FFTs in the transmitter and receiver sections of the radio. One of the benefits of OFDM is its strength in fighting the adverse effects of multipath propagation with respect to inter-symbol interference in a channel. OFDM is also spectrally efficient because the channels are overlapped and contiguous. The block diagram of OFDM is shown figure 2.20.

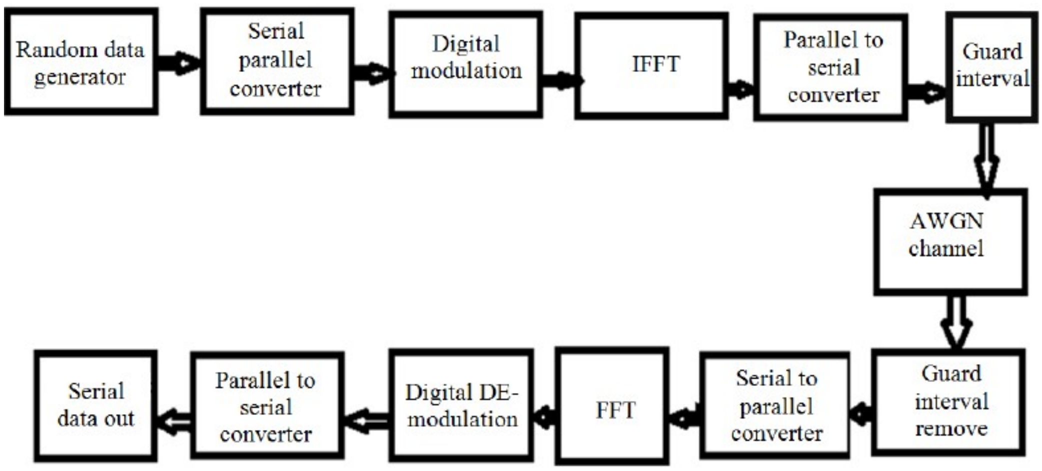


Figure2.20: Block diagram of Orthogonal Frequency Division multiplexing (OFDM)

Random data generator:

The function of random data generator is to generate the random uniform data.

Serial to parallel converter :

The main function of serial to parallel converter is to convert the serial data parallely. The parallel data is transmitted by assigning a unique word to each of the subcarriers. Once the symbol has been allocated to each of the subcarriers then they are phased mapped accordance with modulating scheme.

Digital Modulation:

In the OFDM the different modulation scheme can be applied to each sub channel depends on channel condition, data rate, robustness, throughput and channel bandwidth. There could be different modulation scheme applied i.e., QPSK, 16 QAM. Modulation to OFDM sub channel.

IFFT-Frequency-Domain to Time Domain Conversion:

The orthogonality of subcarrier is maintained and the frequency domain signals are converted into a time domain . In this stage the techniques like IFFT Mapping, Zero mapping and selector bank is included to overcome the problem of length of subcarrier and bin size.

Guard Interval:

When the length of the guard interval is longer than the duration of the channel impulse response, ISI can completely be removed. However, as the transmission efficiency reduces with the insertion of the guard interval .The guard interval must be chosen sufficiently small. The most commonly used guard interval is the Cyclic Prefix.

Parallel to Serial converter:

The final stage in the implementation must undo the first stage. A switch is used to time-division multiplex the four individual bit signals into a single sequence.

AWGN Channel:

Additive white Gaussian Noise (AWGN) is a basic noise model used in Information theory to imitate the effect of many random processes that occur in nature.

FFT: Time Domain to Frequency Domain Conversion:

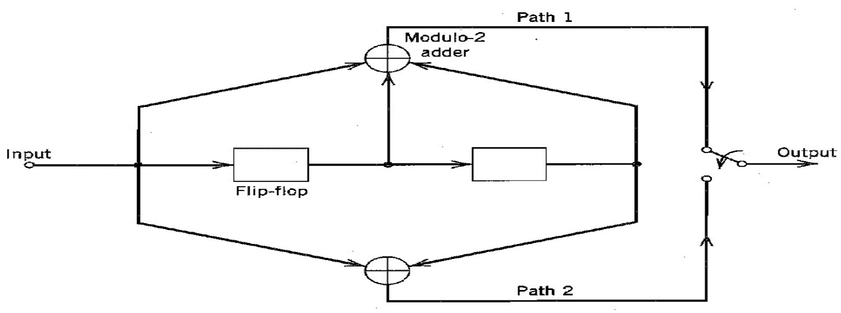
OFDM distributes the data over a large number of carriers at different frequencies. This spacing provides the orthogonality which prevents the receivers to see wrong frequencies. The FFT converts the time-domain to the frequency-domain.

Digital Demodulation:

There could be different demodulation scheme applied i.e., QPSK, 16 QAM. demodulation to OFDM sub channel.

UNIT No V

Q **1:***Figure below shows the encoder for a rate r=1/2 convolution code . Determine the encoder output produced by the message source 10111… , using transform domain approach***.**



Answer:

Code rate= 1/n bits per symbol

M-stage shift register

N- mod 2 adder

L –bit input message

N(L+M) bits output coded sequence

Code rate=r

$$r = \frac{L}{n(L+M)} bits / symbol$$

If L>>M

$$r = \frac{1}{n} \text{bits per symbol}$$

K=constraint length of a convolutional code is defined as the number of shifts over which a

single message bit can influence the encoder output.
=Number of shifts
=M+1

Code rate= $r = \frac{1}{2}$

$r = \frac{input}{output}$

From diagram:
M=2,(Two shift registers)
N=2(Adders=2)
L=5(sequence length)
N(L+M)=2(5+2)=14 =output coded sequence
K=M+1=2+1=3 = constraint length

Manually shifting the input seq.10011

Input	Flipflop1,2 content		Output of top adder,bottom adder
1	1	0	1 1
0	0	1	10
0	0	0	11
1	1	0	11
1	1	1	01

Transform domain Approach:
Step I: Input – Top adder-Output path

$$g^{(1)}(D) = g_0^{(1)} + g_1^{(1)}D + g_2^{(1)}D^2 + \dots\dots\dots\dots + g_M^{(1)}D^M$$

$g_0^{(1)}, g_1^{(1)}, g_2^{(1)}, ...g_M^{(1)}$ =elements of impulse response.

Step II: Input – Top adder-Output path

$g^{(2)}(D) = g_0^{(2)} + g_1^{(2)}D + g_2^{(2)}D^2 + ... + g_M^{(2)}D^M$

$g_0^{(2)}, g_1^{(2)}, g_2^{(2)}, ...g_M^{(2)}$

Message=(m_0,m_1,m_2,m_3,…………………m_{L-1})

Message polynomial=m(D)

$m(D) = m_0 + m_1D + m_2D^2 + ... + m_{L-1}D^{L-1}$

$x^{(1)}(D) = g^{(1)}(D)m(D)$

$x^{(2)}(D) = g^{(2)}(D)m(D)$

For M=2

Top adder impulse response=

$(g_0^{(1)}, g_1^{(1)}, g_2^{(1)}) = (1,1,1)$

$g^{(1)}(D) = g_0^{(1)} + g_1^{(1)}D + g_2^{(1)}D^2$

Bottom adder impulse response=

$(g_0^{(2)}, g_1^{(2)}, g_2^{(2)}) = (1,0,1)$

$g^{(2)}(D) = g_0^{(2)} + g_1^{(2)}D + g_2^{(2)}D^2$

$g^{(1)}(D) = 1 + D + D^2$

$g^{(2)}(D) = 1 + D^2$

M=(1 0 0 1 1)

$m(D) = 1 + 0 \times D + 0 \times D^2 + 1 \times D^3 + 1 \times D^4$

$m(D) = 1 + D^3 + D^4$

$x^{(1)}(D) = g^{(1)}(D)m(D)$

$x^{(1)}(D) = (1 + D + D^2)(1 + D^3 + D^4)$

$x^{(1)}(D) = 1 + D + D^2 + D^3 + D^6$

$x^{(2)}(D) = g^{(2)}(D)m(D)$

$x^{(2)}(D) = (1+D^2)(1+D^3+D^4)$

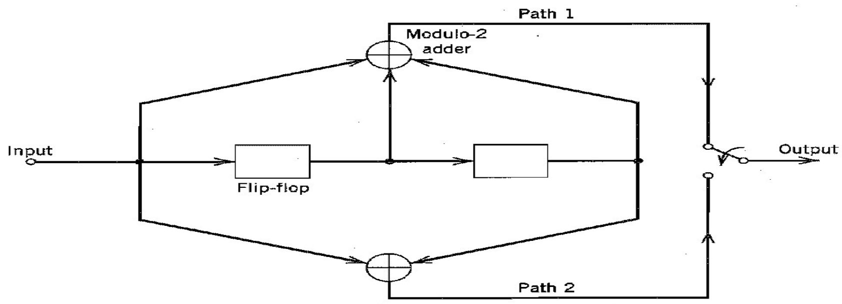
$x^{(2)}(D) = 1+D^2+D^3+D^4+D^5+D^6$

x={11,10,11,11,01,01,11}

n(L+M)=2(5+2)

=14

Q2: *Figure below shows the encoder for a rate r=1/2 convolution code. Determine the encoder output produced by the message source 10111… , using time domain approach.*



Answer:

Top output sequence

$$x_i^{(1)} = \sum_{\substack{l=0 \\ i=0,1,2,...}}^{M} g_l^{(1)} m_{i-l}$$ Where m_{i-l}=0 for all l>i

Bottom output sequence

$$x_i^{(2)} = \sum_{\substack{l=0 \\ i=0,1,2,...}}^{M} g_l^{(2)} m_{i-l}$$

Encoder output sequence: {xi}

$\{x_i\} = \{x_0^{(1)}, x_0^{(2)}, x_1^{(1)}, x_1^{(2)}, x_2^{(1)}, x_2^{(2)} ....\}$

For i=0,M=2(Two shift registers)

$m = (m_0, m_1, m_2, m_3, m_4) = (1, 0, 0, 1, 1)$ =message seq..

Impulse response of input-top adder-output path is $(g_0^{(1)}, g_1^{(1)}, g_2^{(1)}) = (1,1,1)$

Impulse response of input-bottom adder-output path is $(g_0^{(2)}, g_1^{(2)}, g_2^{(2)}) = (1,0,1)$

Top adder	Bottom adder
Top sequence $i = 0, x_0^{(1)} = \sum_{\substack{l=0 \\ i=0}}^{M=2} g_l^{(1)} m_{-l}$ $x_0^{(1)} = g_0^{(1)} m_{-0} + g_1^{(1)} m_{-1} + g_2^{(1)} m_{-2}$ For i=0, $x_0^{(1)} = g_0^{(1)} m_0 = 1$	For i= 0 $x_0^{(2)} = g_0^{(2)} m_{-0} + g_1^{(2)} m_{-1} + g_2^{(2)} m_{-2}$ $x_0^{(2)} = 1$
For i=1, $x_1^{(1)} = \sum_{\substack{l=0 \\ i=1}}^{M=2} g_l^{(1)} m_{-l}$ $x_1^{(1)} = g_0^{(1)} m_1 + g_1^{(1)} m_0 + g_2^{(1)} m_{-1}$ $x_1^{(1)} = g_0^{(1)} m_1 + g_1^{(1)} m_0$ $x_1^{(1)} = 1$	For i=1, $x_1^{(2)} = g_0^{(2)} m_1 + g_1^{(2)} m_0 + g_2^{(2)} m_{-1}$ $x_1^{(2)} = 0$
For i=2, $x_2^{(1)} = \sum_{\substack{l=0 \\ i=2}}^{M=2} g_l^{(1)} m_{-l}$ $x_2^{(1)} = g_0^{(1)} m_2 + g_1^{(1)} m_1 + g_2^{(1)} m_0$ $x_2^{(1)} = 1$	$x_2^{(2)} = g_0^{(2)} m_2 + g_1^{(2)} m_1 + g_2^{(2)} m_0$ $x_2^{(2)} = 1$

For i=3 $x_3^{(1)} = \sum_{\substack{l=0 \\ i=3}}^{M=2} g_l^{(1)} m_{3-l}$ $x_3^{(1)} = g_0^{(1)} m_3 + g_1^{(1)} m_2 + g_2^{(1)} m_1$ $x_3^{(1)} = 1$	$x_3^{(2)} = g_0^{(2)} m_3 + g_1^{(2)} m_2 + g_2^{(2)} m_1$ $x_3^{(1)} = 1$
$x_4^{(1)} = \sum_{\substack{l=0 \\ i=4}}^{M=2} g_l^{(1)} m_{4-l}$ $x_4^{(1)} = g_0^{(1)} m_4 + g_1^{(1)} m_3 + g_2^{(1)} m_2$ $x_4^{(1)} = 0$	$x_4^{(2)} = \sum_{\substack{l=0 \\ i=4}}^{M=2} g_l^{(2)} m_{4-l}$ $x_4^{(2)} = g_0^{(2)} m_4 + g_1^{(2)} m_3 + g_2^{(2)} m_2$ $x_4^{(2)} = 1$
$x_5^{(1)} = \sum_{\substack{l=0 \\ i=5}}^{M=2} g_l^{(1)} m_{5-l}$ $x_5^{(1)} = g_0^{(1)} m_5 + g_1^{(1)} m_4 + g_2^{(1)} m_3$ $x_5^{(1)} = 0$	$x_5^{(2)} = \sum_{\substack{l=0 \\ i=5}}^{M=2} g_l^{(2)} m_{5-l}$ $x_5^{(2)} = g_0^{(2)} m_5 + g_1^{(2)} m_4 + g_2^{(2)} m_3$ $x_5^{(2)} = 1$
$x_6^{(1)} = \sum_{\substack{l=0 \\ i=6}}^{M=2} g_l^{(1)} m_{6-l}$ $x_6^{(1)} = g_0^{(1)} m_6 + g_1^{(1)} m_5 + g_2^{(1)} m_4$ $x_6^{(1)} = 1$	$x_6^{(2)} = \sum_{\substack{l=0 \\ i=6}}^{M=2} g_l^{(2)} m_{6-l}$ $x_6^{(2)} = g_0^{(2)} m_6 + g_1^{(2)} m_5 + g_2^{(2)} m_4$ $x_6^{(2)} = 1$
$x_7^{(1)} = \sum_{\substack{l=0 \\ i=7}}^{M=2} g_l^{(1)} m_{7-l}$ $x_7^{(1)} = g_0^{(1)} m_7 + g_1^{(1)} m_6 + g_2^{(1)} m_5$ $x_7^{(1)} = 0$	$x_7^{(2)} = \sum_{\substack{l=0 \\ i=7}}^{M=2} g_l^{(2)} m_{7-l}$ $x_7^{(2)} = g_0^{(2)} m_7 + g_1^{(2)} m_6 + g_2^{(2)} m_5$ $x_7^{(2)} = 1$

Xi={11,10,11,11,01,01,01,11}

Q3: *Consider the code rate 1/2, constraint length=3, convolutional encoder with the generators.g1=(1, 1, 1),g2=(1, 0, 1),Draw encoder block diagram, code tree, state diagram,trellis diagram.*

Answer:

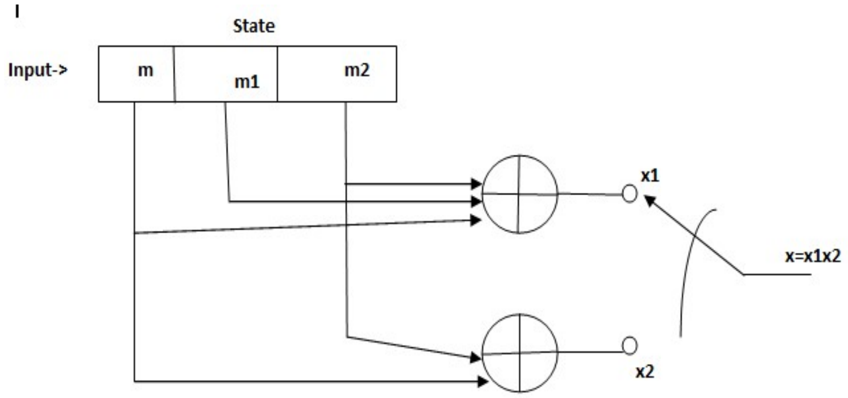
Block diagram:

g1=1 1 1

g2= 1 0 1

$$x_1 = m \oplus m_1 \oplus m_2$$

$$x_2 = m \oplus m_2$$

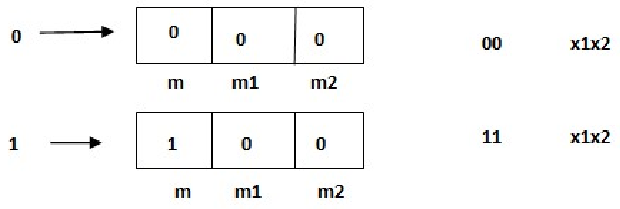


State table

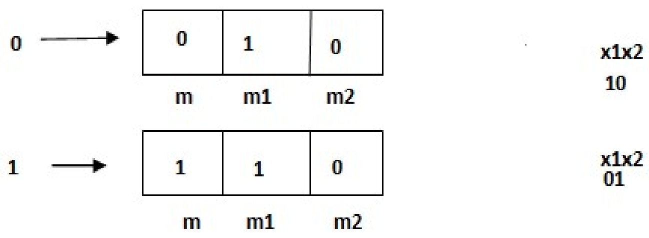
m1	m2	state
0	0	a
1	0	b
0	1	c

1	1	d

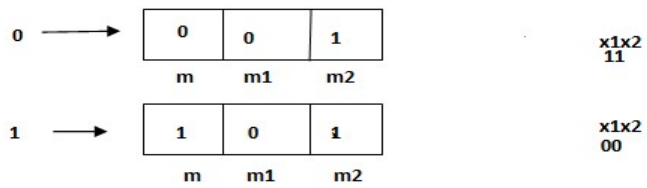
(a)


0
0
0
0
m
m1
m2
00
x1x2
1
1
0
0
m
m1
m2
11
x1x2

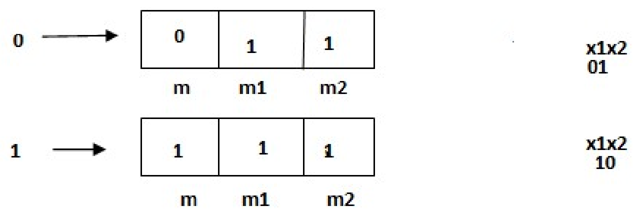
(b)


0
0
1
0
m
m1
m2
x1x2
10
1
1
1
0
m
m1
m2
x1x2
01

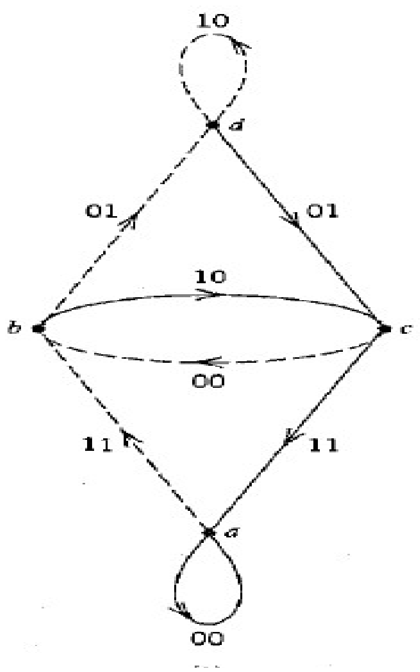
(c)


0
0
0
1
m
m1
m2
x1x2
11
1
1
0
1
m
m1
m2
x1x2
00

(d)



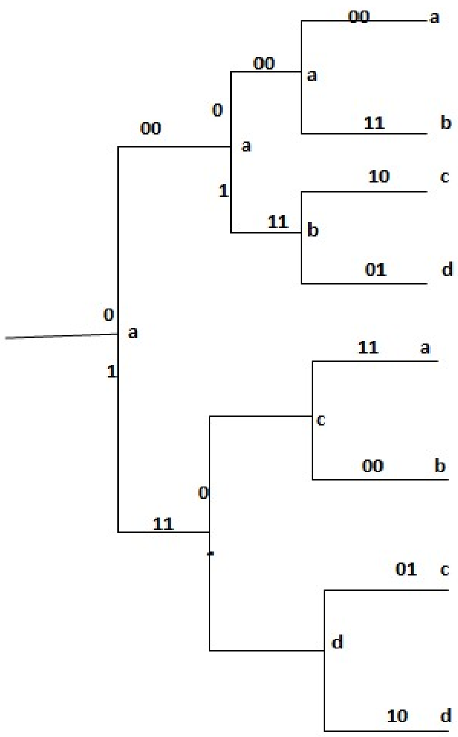
State diagram:



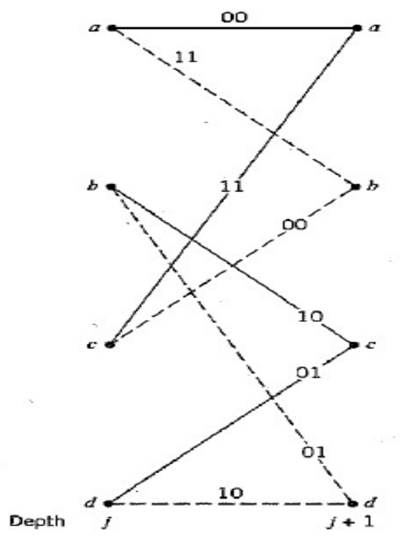
0-solid line

1-dash line

Code tree


00
a
00
a
0
11
b
00
a
1
10
c
11
b
01
d
0
a
1
11
a
c
00
b
0
11
01
c
d
10
d

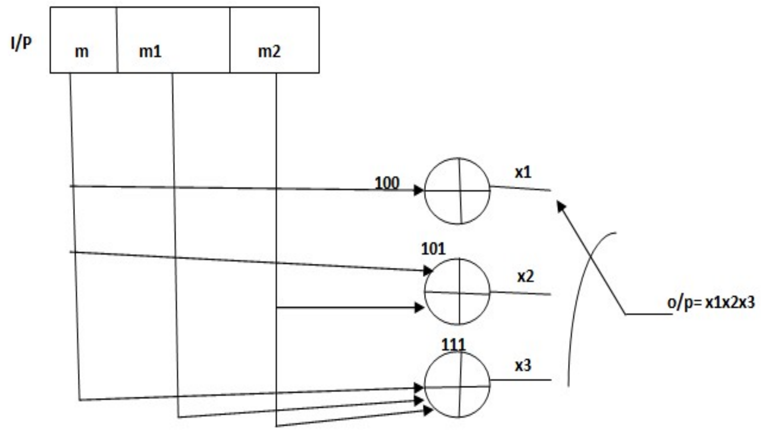
Trellis diagram:



Q4: *Consider the code rate 1/3, convolutional encoder with the generators.*
g1=(1, 0, 0),g2=(1, 0, 1),g3=(1 , 1 , 1)
Draw encoder block diagram, code tree, state diagram,trellis diagram.

Answer:

Block diagram:
g1=1 0 0
g2= 1 0 1
g3=1 1 1



State table

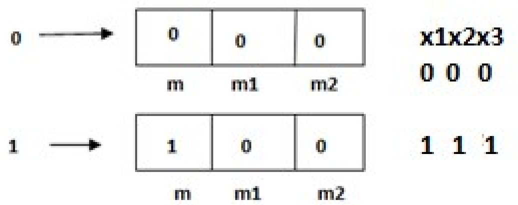
m1	m2	state
0	0	a
1	0	b
0	1	c
1	1	d

$x_1 = m$

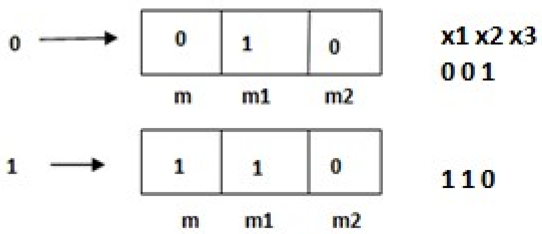
$x_2 = m \oplus m_2$

$x_3 = m \oplus m_1 \oplus m_2$

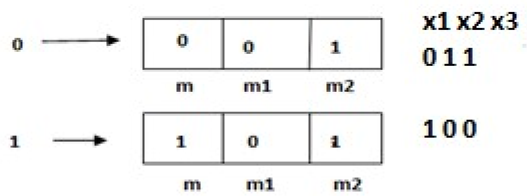
(a)


0
0 0 0
m m1 m2
x1x2x3
0 0 0
1
1 0 0
m m1 m2
1 1 1

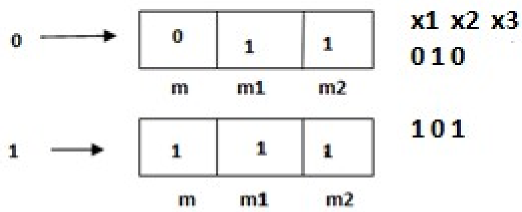
(b)


0
0 1 0
m m1 m2
x1 x2 x3
001
1
1 1 0
m m1 m2
110

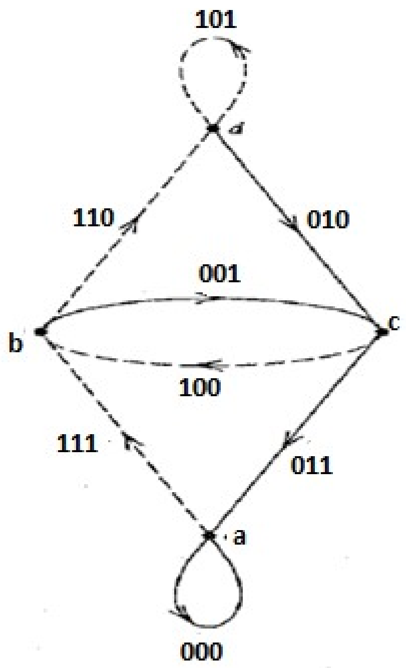
(c)


0
0 0 1
m m1 m2
x1 x2 x3
011
1
1 0 1
m m1 m2
100

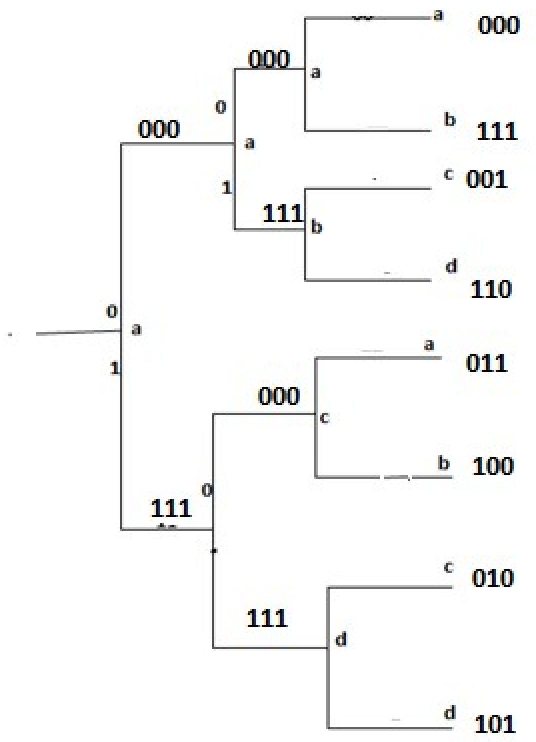
(d)


0
0 1 1
m m1 m2
x1 x2 x3
010
1
1 1 1
m m1 m2
101

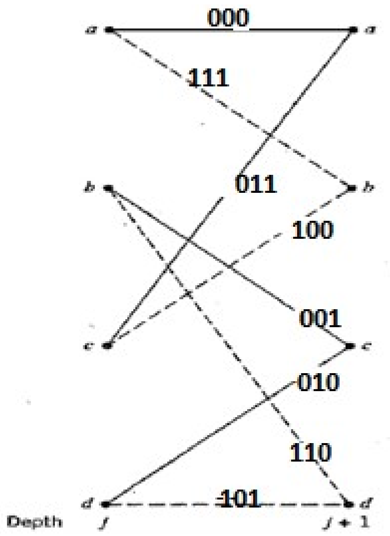
State diagram:



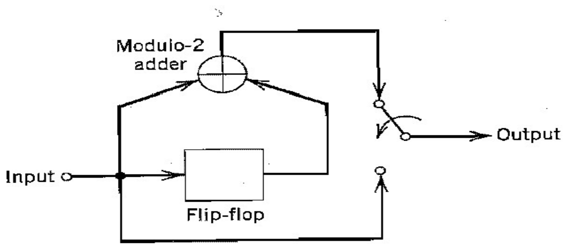
Code tree



Trellis diagram



Q5: *Consider the rate 1/2 , constraint length= 2 , convolutional encoder . The code is systematic. Find the encoder output produced by the message sequence 10111.Use transform domain approach.*



Answer:

By using transform domain approach:

M=1,n=2,L=5,K=M+1=2,n(L+M)=2(5+1)=12

Top adder:

$$g^{(1)}(D)=(g_0{}^{(1)},g_1{}^{(1)})=(1,1)$$

Bottom adder:

$$g^{(2)}(D)=(g_0{}^{(2)},g_1{}^{(2)})=(1,0)$$

$$g^{(1)}(D)=g_0{}^{(1)}+g_1{}^{(1)}D$$

$$g^{(1)}(D)=1+D$$

$$g^{(2)}(D)=g_0{}^{(2)}+g_1{}^{(2)}D$$

$$g^{(2)}(D)=1$$

m=(1 0 1 1 1)

$$m(D)=1+D^2+D^3+D^4$$

Top adder

$$x^{(1)}(D)=g^{(1)}(D)m(D)$$

$$x^{(1)}(D)=(1+D)\times(1+D^2+D^3+D^4)$$

$$x^{(1)}(D)=1+D+D^2+D^5$$

Bottom adder:

$$x^{(2)}(D)=g^{(2)}(D)m(D)$$

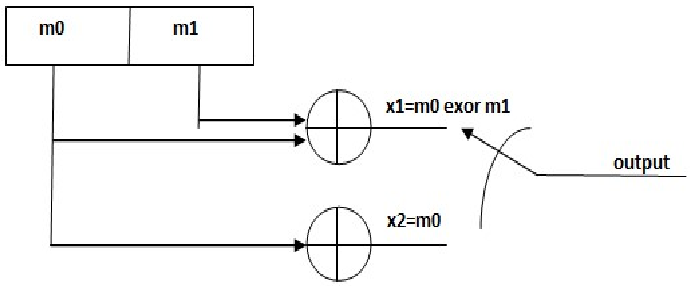
$$x^{(2)}(D) = (1) \times (1 + D^2 + D^3 + D^4)$$

$$x^{(2)}(D) = 1 + D^2 + D^3 + D^4$$

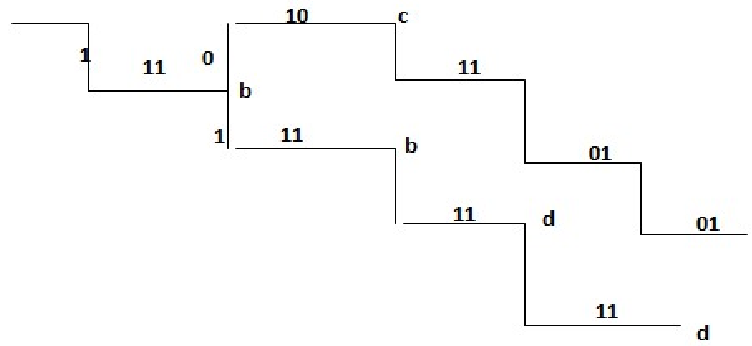
$$x = \{11, 10, 11, 01, 01, 10\}$$

Input	Flipflop	output
1	1	11
0	0	10
1	1	11
1	1	01
1	1	01

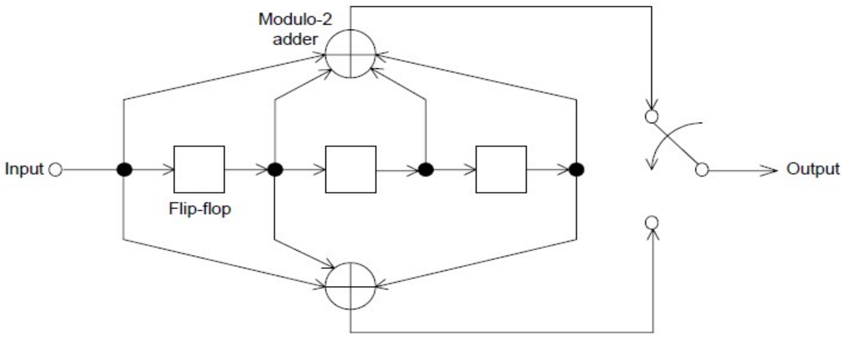
Simplified diagram:



Code tree:



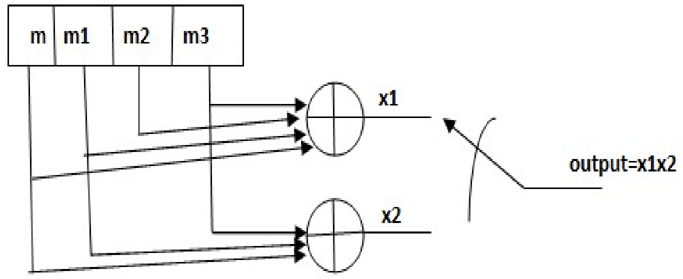
Q6: *Figure shows the encoder for a rate r = 1/2, constraint length K = 4 convolutional code. Determine the encoder output produced by the message sequence 10111 using following Transform domain approach*



Answer:

Simplified diagram:

$$x_1 = m \oplus m_1 \oplus m_2 \oplus m_3$$
$$x_2 = m \oplus m_1 \oplus m_3$$



Message	Flip flop				Output
1	1	0	0	0	11
0	0	1	0	0	11
1	1	0	1	0	01
1	1	1	0	1	11
1	1	1	1	0	10

(b) Transform domain approach:M=3,n=2,L=5,K=4,

n(L+M)=2(5+3)=16

$$g^{(1)}(D) = (g_0^{(1)}, g_1^{(1)}, g_2^{(1)}, g_3^{(1)}) = (1,1,1,1)$$

$g^{(2)}(D) = (g_0{}^{(2)}, g_1{}^{(2)}, g_2{}^{(2)}, g_3{}^{(2)}) = (1,1,0,1)$

I/P-top adder-O/P path

$g^{(1)}(D) = g_0{}^{(1)} + g_1{}^{(1)}D + g_2{}^{(1)}D^2 + g_3{}^{(1)}D^3.$

$g^{(1)}(D) = 1 + D + D^2 + D^3$

I/P-bottom adder-O/P path

$g^{(2)}(D) = g_0{}^{(2)} + g_1{}^{(2)}D + g_2{}^{(2)}D^2 + g_3{}^{(2)}D^3$

$g^{(2)}(D) = 1 + D + D^3$

m=1 0 1 1 1= $m(D) = 1 + D^2 + D^3 + D^4$

Top adder

$x^{(1)}(D) = g^{(1)}(D)m(D)$

$x^{(1)}(D) = (1 + D + D^2 + D^3) \times (1 + D^2 + D^3 + D^4)$
$x^{(1)}(D) = 1 + D + D^3 + D^4 + D^5 + D^7$

Bottom adder:

$x^{(2)}(D) = g^{(2)}(D)m(D)$

$x^{(2)}(D) = (1 + D + D^3) \times (1 + D^2 + D^3 + D^4)$

$x^{(2)}(D) = 1 + D + D^2 + D^3 + D^6 + D^7$

x={11,11,01,11,10,10,01,11}

Input	Flipflop	output
1	100	11
0	010	11
1	101	01
1	110	11
1	111	10

Q7: *Figure shows the encoder for a rate r = 1/2, constraint length K = 4 convolutional code. Determine the encoder output produced by the message sequence 10111 using following Time domain approach*

Answer:

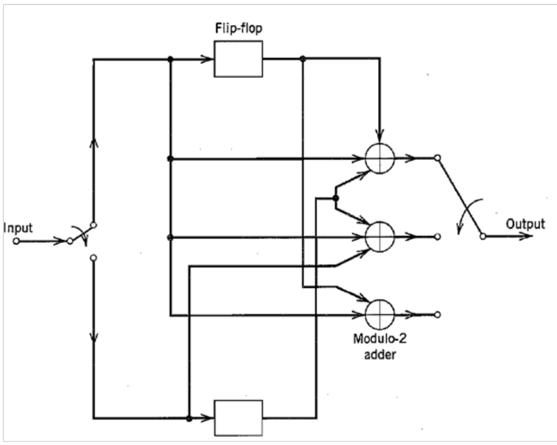
Top adder	Bottom adder
Top sequence $i = 0, x_0^{(1)} = \sum_{\substack{l=0 \\ i=0}}^{M=3} g_l^{(1)} m_{-l}$ $x_0^{(1)} = g_0^{(1)} m_0 + g_1^{(1)} m_{-1} + g_2^{(1)} m_{-2} + g_3^{(1)} m_{-3}$ $x_0^{(1)} = 1$	For i= 0 $x_0^{(2)} = g_0^{(2)} m_0 + g_1^{(2)} m_{-1} + g_2^{(2)} m_{-2} + g_3^{(2)}$ $x_0^{(2)} = 1$

For i=1, $x_1^{(1)} = g_0^{(1)} m_1 + g_1^{(1)} m_0 + g_2^{(1)} m_{-1} + g_3^{(1)} m_{-2}$ $x_1^{(1)} = 1$	For i=1, $x_1^{(2)} = g_0^{(2)} m_1 + g_1^{(2)} m_0 + g_2^{(2)} m_{-1} + g_3^{(2)}$ $x_1^{(2)} = 1$
For i=2, $x_2^{(1)} = g_0^{(1)} m_2 + g_1^{(1)} m_1 + g_2^{(1)} m_0 + g_3^{(1)} m_{-1}$ $x_2^{(1)} = 0$	$x_2^{(2)} = g_0^{(2)} m_2 + g_1^{(2)} m_1 + g_2^{(2)} m_0 + g_3^{(2)}$ $x_2^{(2)} = 1$
For i=3 $x_3^{(1)} = g_0^{(1)} m_3 + g_1^{(1)} m_2 + g_2^{(1)} m_1 + g_3^{(1)} m_0$ $x_3^{(1)} = 1$	$x_3^{(2)} = g_0^{(2)} m_3 + g_1^{(2)} m_2 + g_2^{(2)} m_1 + g_3^{(2)}$ $x_3^{(2)} = 1$
$x_4^{(1)} = g_0^{(1)} m_4 + g_1^{(1)} m_3 + g_2^{(1)} m_2 + g_3^{(1)} m_1$ $x_4^{(1)} = 1$	$x_4^{(2)} = g_0^{(2)} m_4 + g_1^{(2)} m_3 + g_2^{(2)} m_2 + g_3^{(2)}$ $x_4^{(2)} = 0$
$x_5^{(1)} = g_0^{(1)} m_5 + g_1^{(1)} m_4 + g_2^{(1)} m_3 + g_3^{(1)} m_2$ $x_5^{(1)} = 0$	$x_5^{(2)} = g_0^{(2)} m_5 + g_1^{(2)} m_4 + g_2^{(2)} m_3 + g_3^{(2)}$ $x_5^{(2)} = 0$
$x_6^{(1)} = g_0^{(1)} m_6 + g_1^{(1)} m_5 + g_2^{(1)} m_4 + g_3^{(1)} m_3$ $x_6^{(1)} = 0$	$x_6^{(2)} = g_0^{(2)} m_6 + g_1^{(2)} m_5 + g_2^{(2)} m_4 + g_3^{(2)}$ $x_6^{(2)} = 1$
$x_7^{(1)} = g_0^{(1)} m_7 + g_1^{(1)} m_6 + g_2^{(1)} m_5 + g_3^{(1)} m_4$ $x_7^{(1)} = 1$	$x_7^{(2)} = g_0^{(2)} m_7 + g_1^{(2)} m_6 + g_2^{(2)} m_5 + g_3^{(2)}$ $x_7^{(2)} = 1$

$x_8^{(1)} = g_0^{(1)} m_8 + g_1^{(1)} m_7 + g_2^{(1)} m_6 + g_3^{(1)} m_5$ $x_8^{(1)} = 0$	$x_8^{(2)} = g_0^{(2)} m_8 + g_1^{(2)} m_7 + g_2^{(2)} m_6 + g_3^{(2)}$ $x_8^{(2)} = 0$

X={11,11,01,11,10,10,01,11,00}

Q8:*Consider the encoder diagram given below for a rate r=2/3, k=2, convolutional code using the transform domain approach determine the code sequence produced by the message sequence 10111.*



Answer:

Upper flipflop is connected to following adders

$$g_0^{(1)} = (1,1) = 1 + D$$
$$g_1^{(1)} = (1,0) = 1$$
$$g_2^{(1)} = (1,1) = 1 + D$$

Lower flip flop is connected to following adders

$g_0^{(2)} = (0,1) = D$
$g_1^{(2)} = (1,1) = 1 + D$
$g_2^{(2)} = (0,0) = 0$

The message sequence is 10111

$m^{(1)}$=1 1 1 = 1+D+D^2

$m^{(2)}$=0 1 0 = D

Output sequence is

$C(0) = g_0^{(1)} m^{(1)} + g_0^{(2)} m^{(2)}$
$C(0) = (1 + D)(1 + D + D^2) + D(D)$

$C(0) = 1 + D^2 + D^3$ =>(1 0 1 1)

$C(1) = g_1^{(1)} m^{(1)} + g_1^{(2)} m^{(2)}$
$C(1) = (1)(1 + D + D^2) + (1 + D)(D)$

$C(1) = 1$ =>(1 0 0 0)

$C(2) = g_2^{(1)} m^{(1)} + g_2^{(2)} m^{(2)}$
$C(2) = (1 + D)(1 + D + D^2) + 0$

$C(2) = 1 + D^3$ =>(1 0 0 1)

C(0)=(1 0 1 1)

C(1)=(1 0 0 0)

C(2)=(1 0 0 1)

Output= x={111,000,100,101}

Q 9: *The parity check matrix of (6,3) linear block code is given below. Find all the code vectors of this code*

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Answer:

N=6, k=3,q=n-k=6-3=3

$$I=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad P=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

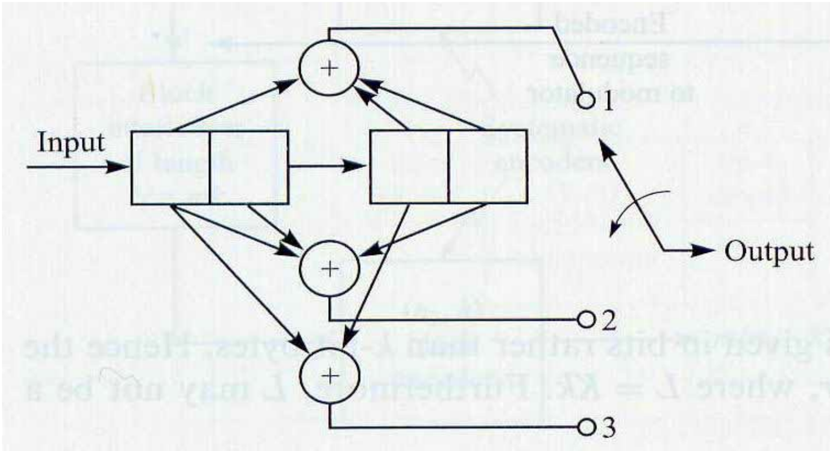
C=MP

$$[C1\ \ C2\ C3]_{1\times 3}=[m1\ m2\ m3]_{1\times 3} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

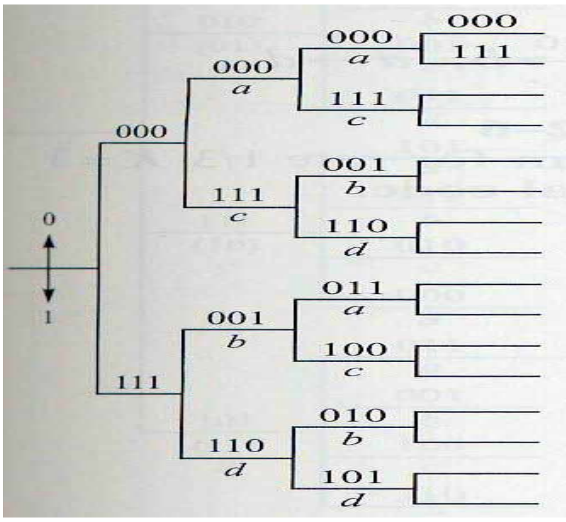
Bits of message vector m1m2m3			Check bits C1C2C3			Complete codeword m1m2m3C1C2C3					
0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	0	0	0	1	1	1	0
0	1	0	1	0	1	0	1	0	1	0	1
0	1	1	0	1	1	0	1	1	0	1	1
1	0	0	0	1	1	1	0	0	0	1	1
1	0	1	1	0	1	1	0	1	1	0	1
1	1	0	1	1	0	1	1	0	1	1	0
1	1	1	0	0	0	1	1	1	0	0	0

Q10: *Consider a rate 2/3 convolutional encoder.*
The generators are: g1=[1011], g2=[1101], and g3=[1010]. Draw the block diagram.Draw tree ,trellis ,state diagram

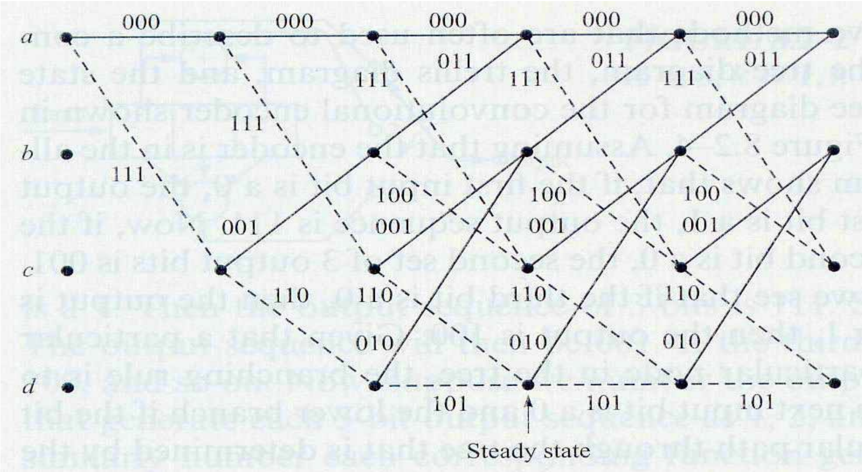
Answer:



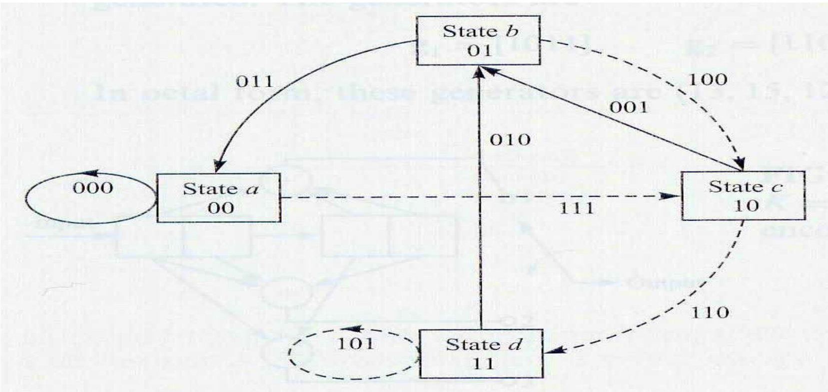
Block diagram of 2/3 convolutional encoder



Tree diagram



Trellis diagram



State diagram

$a \xrightarrow{0} a \quad a \xrightarrow{1} c$

$b \xrightarrow{0} a \quad b \xrightarrow{1} c$

$c \xrightarrow{0} b \quad c \xrightarrow{1} d$

$d \xrightarrow{0} b \quad d \xrightarrow{1} d$

UNIT No. VI

SPREAD-SPECTRUM MODULATION

Q1. *Consider a maximal length sequence requiring the use of linear feedback shift register of length m=4 as shown in figure 6.1. For feedback tabs select the set [4,3]. The initial state of the register is [1000].Generate the PN sequence for given initial states of register.*

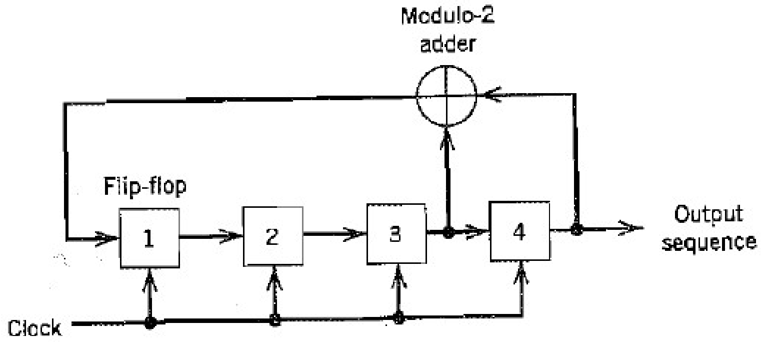


Figure 6.1: Maximum length sequence generator with feedback tabs [4,3]

Answer:

The initial state of the register is [1000].

There will be $(2^m-1) = (2^4-1) = 15$ shifts

Modulo-2 adder output= state 3 xor state 4

Table 1: Evolution of maximum length sequence generator with feedback shift register [4,3]

Shift Number	Shift register contents	Modulo-2 adder output	Shift register output
0	1000	0 xor 0=0	--
1	0100	0 xor 0=0	0
2	0010	0 xor 0=0	0
3	1001	1 xor 0=1	0
4	1100	0 xor 1=1	1
5	0110	1 xor 0=1	0
6	1011	1 xor 1=0	0
7	0101	0 xor 1=1	1
8	1010	1 xor 0=1	1
9	1101	0 xor 1=1	0
10	1110	1 xor 0=1	1
11	1111	1 xor 1=0	0
12	0111	1 xor 1=0	1
13	0011	1 xor 1=0	1
14	0001	0 xor 1=1	1
15	1000	0 xor 0=0	1

The output PN sequence =000100110101111

Q2. *Consider a maximal length sequence requiring the use of linear feedback shift register of length m=5 as shown in figure 6.2. For feedback tabs select the set [2,5]. The initial state of the register is [1000].Generate the PN sequence for given initial states of register.*

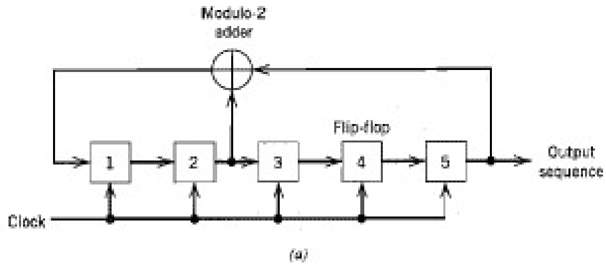


Figure 6.2: Maximum length sequence generator with feedback tabs [2,5]

Answer:

The initial state of the register is [10000].

There will be (2^m-1) $=(2^5-1)=31$ shifts

Modulo-2 adder output= state 2 xor state 5

Table 2: Evolution of maximum length sequence generator with feedback shift register [5,2]

Shift Number	**Shift register contents**	**Modulo-2 adder output**	**Shift register output**
0	10000	0 xor 0=0	---
1	01000	1 xor 0=1	0
2	10100	0 xor 0=0	0
3	01010	1 xor 0=1	0
4	10101	0 xor 1=1	0
5	11010	1 xor 0=1	1
6	11101	1 xor 1=0	0
7	01110	1 xor 0=1	1
8	10111	0 xor 1=1	0
9	11011	1 xor 1=0	1
10	01101	1 xor 1=0	1
11	00110	0 xor 0=0	1
12	00011	0 xor 1=1	0
13	10001	0 xor 1=1	1
14	11000	1 xor 0=1	1
15	11100	1 xor 0=1	0

16	11110	1 xor 0=1	0
17	11111	1 xor 1=0	0
18	01111	1 xor 1=0	1
19	00111	0 xor 1=1	1
20	10011	0 xor 1=1	1
21	11001	1 xor 1=0	1
22	01100	1 xor 0=1	1
23	10110	0 xor 0=0	0
24	01011	1 xor 1=0	0
25	00101	0 xor 1=1	1
26	10010	0 xor 0=0	1
27	01001	1 xor 1=0	0
28	00100	0 xor 0=0	1
29	00010	0 xor 0=0	0
30	00001	0 xor 1=1	0
31	10000	0 xor 0=0	1

The output PN sequence =0000101011101100011111001101001

Q3. *Consider a maximal length sequence requiring the use of linear feedback shift register of length m=5 as shown in figure 6.3. For feedback tabs select the set [5,4,2,1]. The initial state of the register is [1000].Generate the PN sequence for given initial states of register.*

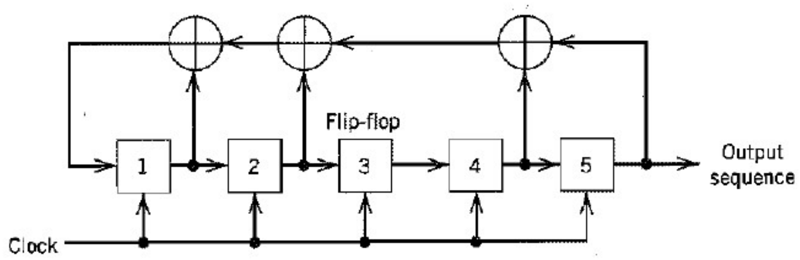


Figure 6.3: Maximum length sequence generator with feedback tabs [5,4,2,1]

Answer:

The initial state of the register is [10000].

There will be $(2^m-1) = (2^5-1) = 31$ shifts

Modulo-2 adder output= state 1 xor state 2 xor state 4 xor state 5

Table 3: Evolution of maximum length sequence generator with feedback shift register [5,4,2,1]

Shift Number	Shift register contents	Modulo-2 adder output	Shift register output
0	10000	1 xor 0 xor 0 xor 0=1	---
1	11000	1 xor 1 xor 0 xor 0=0	0
2	01100	0 xor 1 xor 0 xor 0=1	0
3	10110	1 xor 0 xor 1 xor 0=0	0
4	01011	0 xor 1 xor 1 xor 1=1	0
5	10101	1 xor 0 xor 0 xor 1=0	1
6	01010	0 xor 1 xor 1 xor 0=0	1
7	00101	0 xor 0 xor 0 xor 1=1	0
8	10010	1 xor 0 xor 1 xor 0=0	1
9	01001	0 xor 1 xor 0 xor 1=0	0
10	00100	0 xor 0 xor 0 xor 0=0	1
11	00010	0 xor 0 xor 1 xor 0=1	0
12	10001	1 xor 0 xor 0 xor 1=0	0
13	01000	0 xor 1 xor 0 xor 0=1	1

14	10100	1 xor 0 xor 0 xor 0=1	0
15	11010	1 xor 1 xor 1 xor 0=1	0
16	11101	1 xor 1 xor 0 xor 1=1	0
17	11110	1 xor 1 xor 1 xor 0=1	1
18	11111	1 xor 1 xor 1 xor 1=0	0
19	01111	0 xor 1 xor 1 xor 1=1	1
20	10111	1 xor 0 xor 1 xor 1=1	1
21	11011	1 xor 1 xor 1 xor 1=0	1
22	01101	0 xor 1 xor 0 xor 1=0	1
23	00110	0xor 0 xor 1 xor 0=1	1
24	10011	1 xor 0 xor 1 xor 1=1	0
25	11001	1 xor 1 xor 0 xor 1=1	1
26	11100	1 xor 1xor 0 xor 0=0	1
27	01110	0 xor 1 xor 1 xor 0=0	0
28	00111	0 xor 0 xor 1 xor 1=0	0
29	00011	0 xor 0 xor 1 xor 1=0	1
30	00001	0 xor 0 xor 0xor 1=1	1
31	10000	1 xor 0 xor 0 xor 0=1	1

The output PN sequence

=0000110101001000101111011100111

Q4. *Consider a maximal length sequence requiring the use of linear feedback shift register of length m=4 as shown in figure 6.4. For feedback tabs select the set [4,1]. The initial state of the register is [1100].Generate the PN sequence for given initial states of register.*

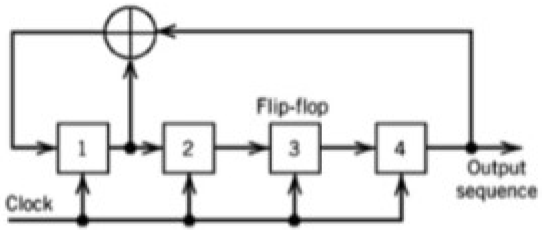


Figure 6.4: Maximum length sequence generator with feedback tabs [4,1]

Answer:

The initial state of the register is [1000].

There will be $(2^m-1) = (2^4-1) = 15$ shifts

Modulo-2 adder output= state 1 xor state 4

Table 4: Evolution of maximum length sequence generator with feedback shift register [4,1]

Shift Number	**Shift register contents**	**Modulo-2 adder output**	**Shift register output**
0	1100	1 xor 0=1	--
1	1110	1 xor 0=1	0
2	1111	1 xor 1=0	0
3	0111	0 xor 1=1	1
4	1011	1 xor 1=0	1
5	0101	1 xor 0=1	1
6	1010	0 xor 1=1	1
7	1101	1 xor 1=0	0
8	0110	0 xor 0=0	1
9	0011	0 xor 1=1	0
10	1001	1 xor 1=0	1
11	0100	0 xor 0=0	1
12	0010	0 xor 0=0	0
13	0001	0 xor 1=1	0
14	1000	1 xor 0=1	1
15	1100	1 xor 0=1	0

The output PN sequence =001111010110010

Q5. *Consider linear feedback shift register involving three flip flops i.e, m=3 as shown in figure 6.5. For feedback tabs select the set [3,1]. Assume that initial state of the register is [100].Generate the PN sequence for given initial states of register.*

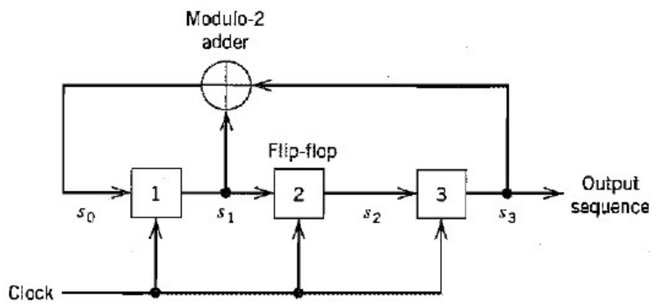


Figure 6.5: Maximum length sequence generator with feedback tabs [3,1]

Answer:

The initial state of the register is [100].

There will be $(2^m-1) = (2^3-1) = 7$ shifts

Modulo-2 adder output= state 1 xor state 3

Table 5: Evolution of maximum length sequence generator with feedback shift register [3,1]

Shift Number	Shift register contents	Modulo-2 adder output	Shift register output
0	100	1 xor 0=1	--
1	110	1 xor 0=1	0
2	111	1 xor 1=0	0
3	011	0 xor 1=1	1
4	101	1 xor 1=0	1
5	010	0 xor 0=0	1
6	001	0 xor 1=1	0
7	100	1 xor 0=1	1

The output PN sequence =0011101

Q6. Explain Direct Sequence Spread Spectrum (DSSS) with transmitter and receiver block diagram.

Answer:

Direct Sequence Spread Spectrum (DSSS) Transmitter:

Transmitter of DSSS shown in figure 6.6 first converts the incoming binary sequence b_k into a polar NRZ waveform b(t) which is followed by two stages of modulation. The first stage consists of a product modulator or multiplier with the data signal b(t) and the PN signal c(t) as inputs. The second stage consists of a binary PSK modulator. The transmitted signal x(t) is the direct sequence spread binary phase shift keying(DS/BPSK)signal.The phase modulation $\varphi(t)$ of x(t) has one of two values π 0r 0, depending on the polarities of the message signal b(t) and PN signal c(t) accordance with the truth table shown in Table 6. Figure 6.7 shows illustrates the waveforms for the second stage of modulation .

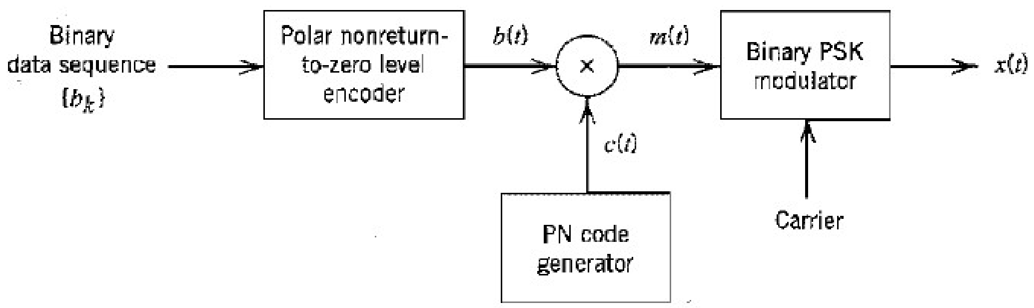


Figure 6.6: Direct Sequence Spread Spectrum (DSSS) Transmitter

Table 6: Truth table for phase modulation $\varphi(t)$,radians

		Polarity of Data sequence b(t) at time t	
		+	-
Polarity of PN sequence c(t) at time t	+	0	π
	-	Π	0

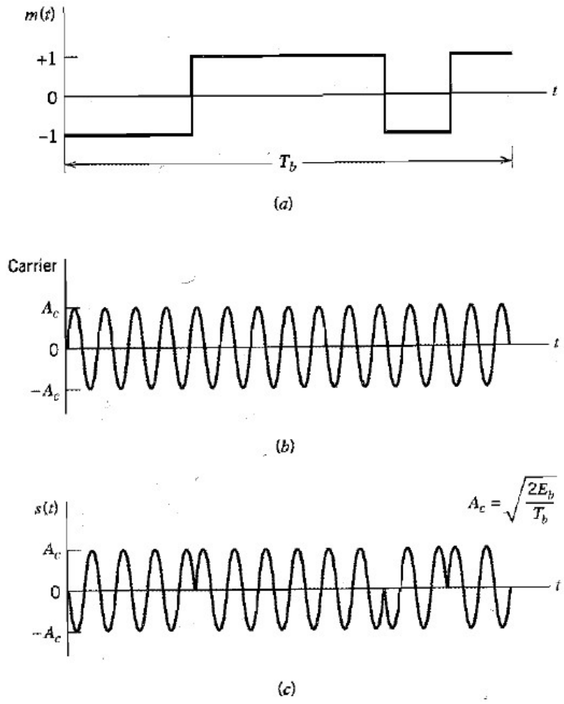


Figure 6.7: (a) Product signal m(t)=c(t)b(t) (b) sinusoidal Carrier (c) DS/BPSK

Direct Sequence Spread Spectrum (DSSS) Receiver:

The receiver shown in figure 6.8, consists of two stages of demodulation. In the first stage the received signal y(t) and a locally generated carrier is applied to modulator followed by low pass filter whose bandwidth is equal to that of the original message signal m(t). This stage of the demodulation process reverses the phase shift keying applied to the transmitted signal. The second stage of demodulation performs spectrum dispreading by multiplying the low pass filter output by a locally generated replica of PN signal c(t) followed by integrator over an interval of $0 \leq t \leq T_b$ providing sample output v. If v > 0 then symbol 1 is transmitted and if v < 0 then symbol 0 is transmitted, if v=0 then receiver makes a random decision 1 or 0.

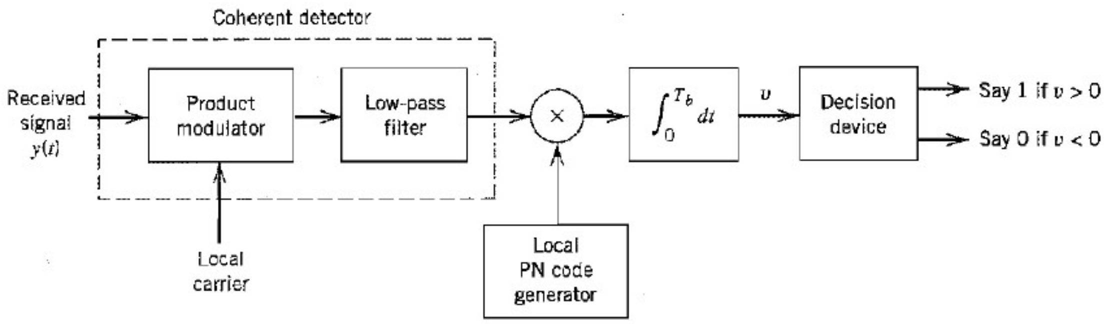


Figure 6.8: Direct Sequence Spread Spectrum (DSSS) Receiver

Q7. Describe a model of Direct Sequence Spread binary PSK system.

Answer:

The model Direct Sequence Spread Spectrum shown in figure 6.9 consists of transmitter-channel-receiver. In this model assumed that the interference j(t) limits the performance so that effect of channel noise may be ignored.

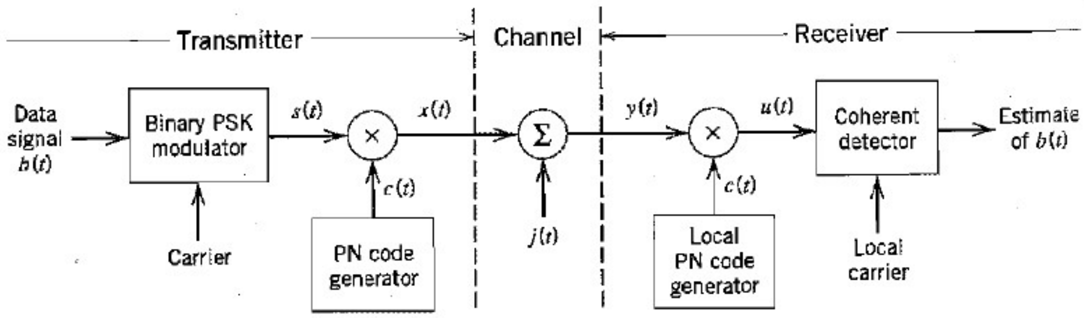


Figure 6.9 : A model of Direct Sequence Spread binary PSK system

Accordingly the channel output is given as:

$$y(t) = x(t) + j(t)$$

$$= c(t) * s(t) + j(t) \qquad (1)$$

Where s(t) is the BPSK signal and c(t) is the PN signal, j(t) interference signal.

In the receiver the received signal y(t) is first multiply by PN signal c(t) yielding output u(t) which is given as an input to the coherent detector. Thus u(t) is given as:

$$u(t) = c(t) * y(t)$$

$$= c(t) * (c(t) * s(t) + j(t))$$

$$= c^2(t) * s(t) + c(t) * j(t)$$

$$= s(t) + c(t) * j(t) \qquad (2)$$

Since the PN sequence satisfies property : $c^2(t) = 1$ for all t.

Equation 2 shows that coherent detector input u(t) consists of BPSK signal s(t) and interference c(t) * j(t).

***Q8.** Explain frequency hopping spread spectrum. Illustrate the concepts of Slow frequency.*

Answer:

The type of spread spectrum in which carrier hops randomly from one frequency to another is called as Frequency Hopping Spread Spectrum(FHSS).Two basic characterization of frequency hopping is Slow frequency hopping and fast frequency hopping.

Slow Frequency hopping--Symbol rate R_s of the MFSK signal is an integral multiple of hop rate Rh, i.e. several symbols are transmitted on each hop(Rh> R_s).

FH/MFSK-Transmitter

Figure 6.10 shows the block diagram of an FH/MFSK transmitter, which involves frequency modulation followed by mixing .First the incoming binary data are applied to an M-ary FSK modulator. The resulting modulated wave and output from a digital frequency synthesizer are then applied to mixer that consists of a multiplier followed by a filter. The filter is designed to select the sum frequency component resulting from the multiplication process as the transmitted signal. The resulting output of the transmitter is FH/MFSK.

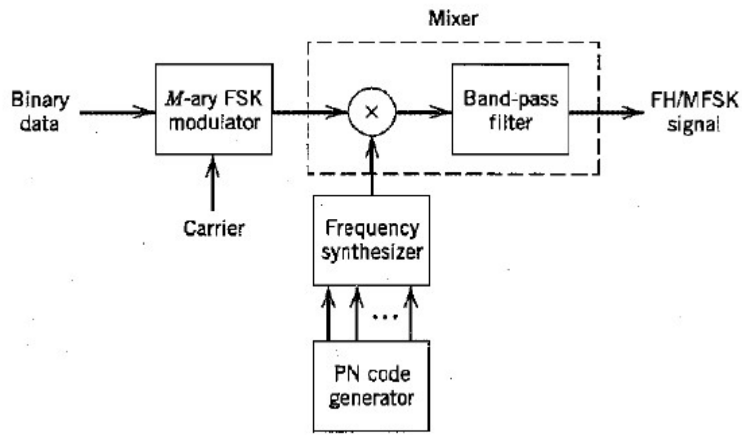


Figure 6.10: FH/MFSK-Transmitter

FH/MFSK Receiver

In the FH/MFSK receiver shown in figure 6.11, the frequency hopping is first removed by mixing the received signal with the output of local frequency synthesizer same as in the transmitter. The resulting output is then band pass filtered and subsequently processed by a non coherent M-ary FSK detector. An estimate of the original symbol transmitted is obtained at the filter output.

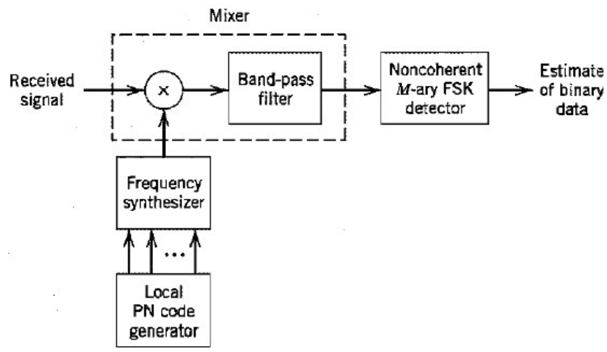


Figure 6.11: FH/MFSK-Receiver

Example of slow frequency hopping with following parameters is shown in the figure 6.12.

The FH/MFSK parameters has the following parameters:

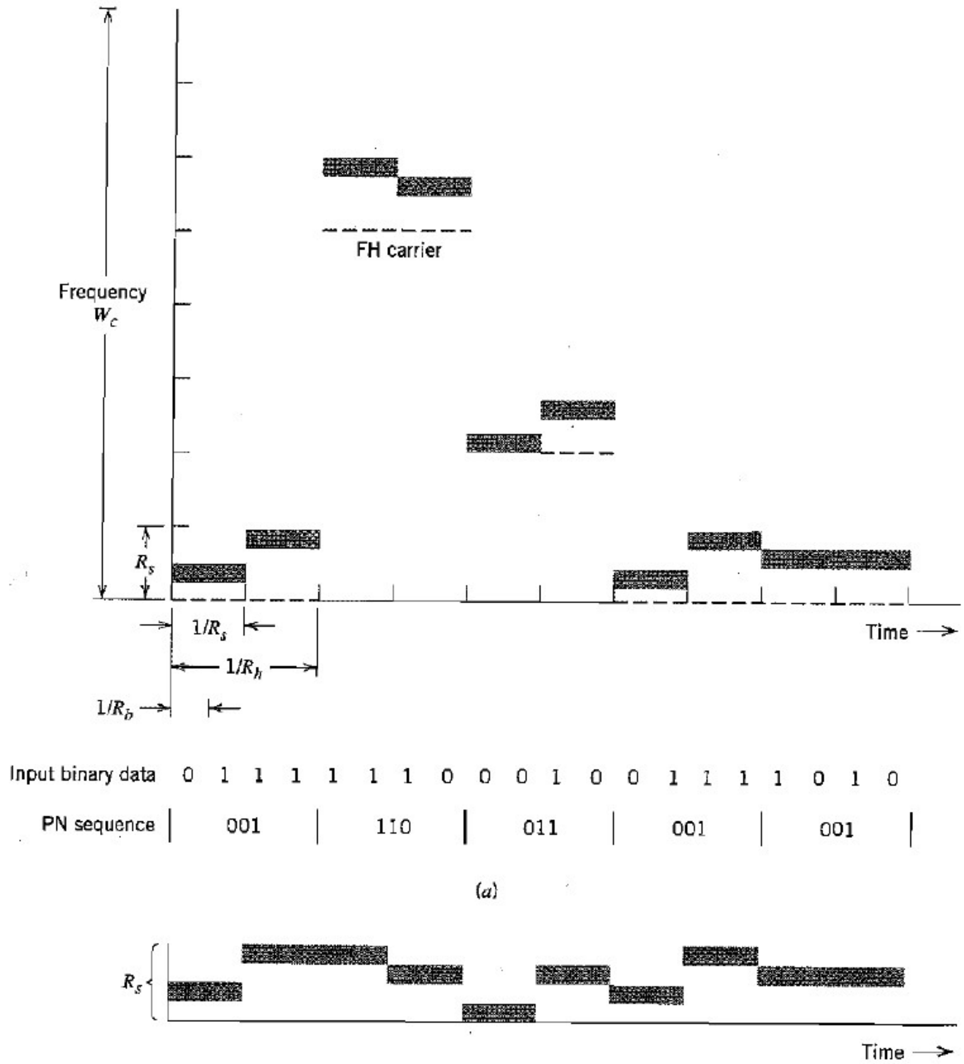
Number of bits per MFSK symbol K=2 ,

Number of MFSK tones M= 2^K=4

Length of PN segment per hop=k=3

Total number of frequency hops=2^k =8

The period of the PN sequence =2^4-1=15



(a)

Figure 6.12: Slow frequency hopping

***Q9**. Explain frequency hopping spread spectrum. Illustrate the concepts of Fast frequency.*

Answer:

The type of spread spectrum in which carrier hops randomly from one frequency to another is called as Frequency Hopping Spread Spectrum(FHSS).Two basic characterization of frequency hopping is Slow frequency hopping and fast frequency hopping.

Fast Frequency hopping- Hop rate Rh, is an integral multiple of Symbol rate R_s of the MFSK signal, i.e.the carrier frequency will change or hop several times during the transmission of one symbol.

FH/MFSK-Transmitter

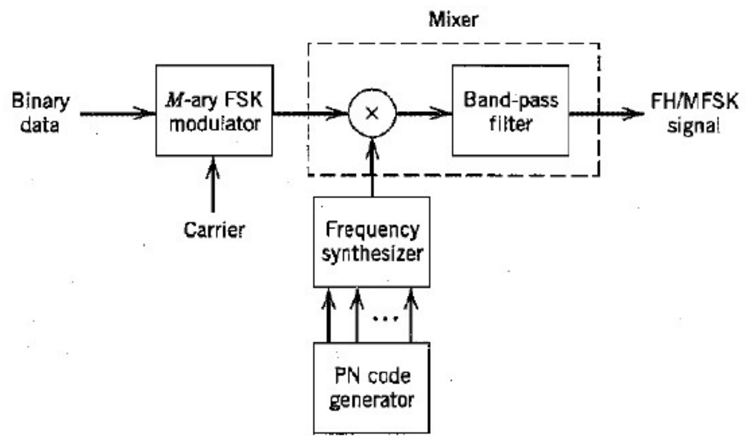


Figure 6.13: FH/MFSK-Transmitter

Figure 6.13 shows the block diagram of an FH/MFSK transmitter, which involves frequency modulation followed by mixing .First the incoming binary data are applied to an M-ary FSK modulator. The resulting modulated wave and output from a digital frequency synthesizer are then applied to mixer that consists of a multiplier followed by a filter. The filter is designed to select the sum frequency component resulting from the multiplication process as the transmitted signal. The resulting output of the transmitter is FH/MFSK.

FH/MFSK Receiver

In the FH/MFSK receiver shown in figure 6.14, the frequency hopping is first removed by mixing the received signal with the output of local frequency synthesizer same as in the transmitter. The resulting output is then band pass filtered and subsequently processed by a non coherent M-ary FSK detector. An estimate of the original symbol transmitted is obtained at the filter output.

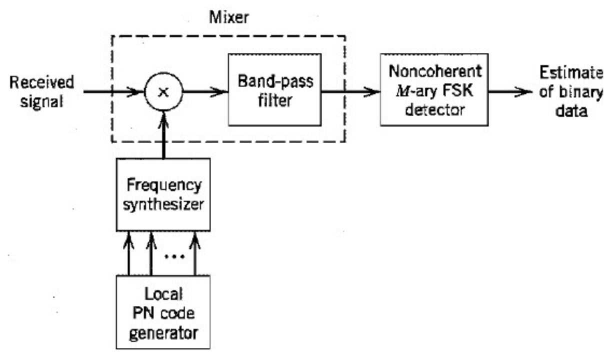


Figure 6.14: FH/MFSK-Receiver

Example of slow frequency hopping with following parameters is shown in the figure 3.

The FH/MFSK parameters has the following parameters:

Number of bits per MFSK symbol K=2 ,

Number of MFSK tones $M= 2^K=4$

Length of PN segment per hop=k=3

Total number of frequency hops=$2^k = 8$

The period of the PN sequence $= 2^4 - 1 = 15$

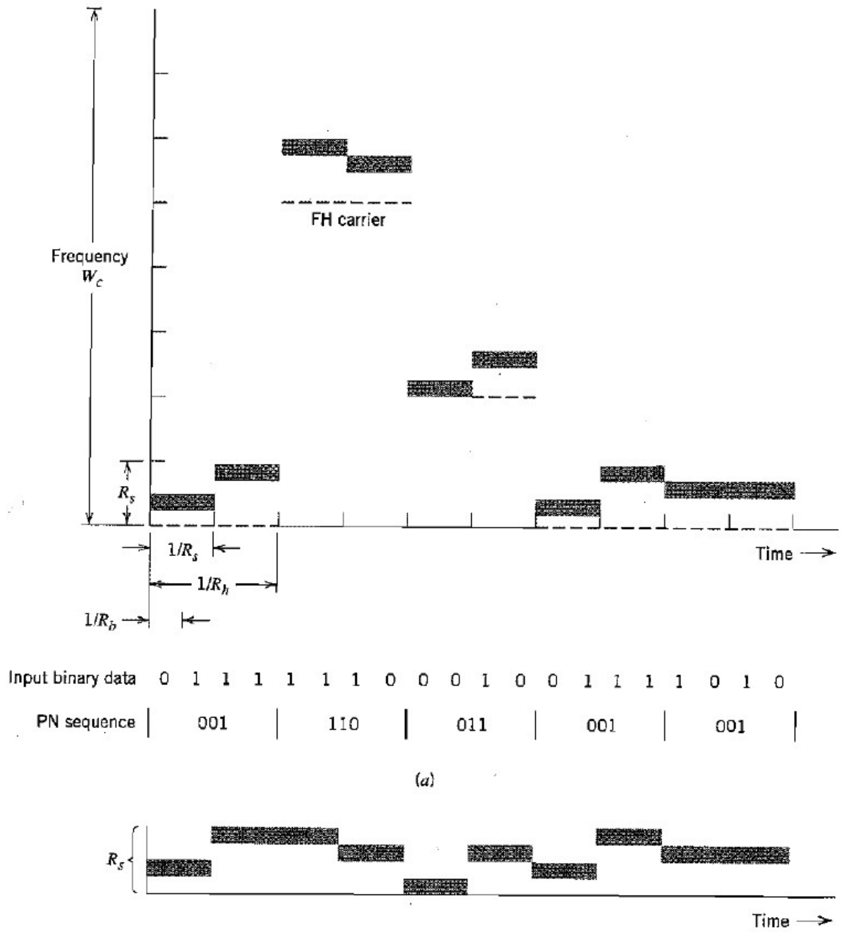


Figure 6.15: Fast Frequency Hopping

Q.10 *Write short note on Code Division Multiple Access.*

Answer:

Code Division Multiple Access:

The two most common multiple access techniques for satellite communications are frequency-division multiple access (FDMA) and time-division multiple access (TDMA), In FDMA, all users access the satellite channel by transmitting simultaneously but using disjoint frequency bands, In TDMA, all users occupy the same RF bandwidth of the satellite channel, but they transmit sequentially in time. When , however, all users are permitted to transmit simultaneously and also occupy the same RF bandwidth of the satellite channel, then some other method must be provided for separating ;the individual signals at the receiver. Code-division multiple access (CDMA) is the method that makes it possible to perform this separation

To accomplish CDMA, spread spectrum is always used. In particular, each user is assigned a code of its own, which performs the direct-sequence or frequency-hop spread-spectrum modulation. The design of the codes has to cater for two provisions:

1. Each code is approximately orthogonal (i.e., has low cross –correlation) with all the other codes.
2. The CDM system operates asynchronously which means that the transition time of users data symbols do not have to coincide with those of the other users.

The second requirement complicates the design of good codes for CDMA. The use of CDM offers three attractive features over TDMA:

1. CDMA does not require an external synchronization network,which is an essential feature of CDMA
2. CDMA offers a gradual degradation in performance as the number users increased .It is therefore relatively easy to add new user to the system.
3. CDMA offers an external interference rejection capability.

References

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[2] Digital and analog communication systems, K. Sam Shanmugam, Singapore : J. Wiley, 5th Edition

[3] Digital Communication,John G Prokis,Springer Publication,3rd Edition

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